COMPLETION OF LINEAR AND NON-LINEAR EQUATIONS USING THE NEW HOMOTOPY PERTUBARTION METHOD

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Abstract:
The intention of this project is to compose advanced process for determine of linear and nonlinear equations. The current techniques is New Homotopy Perturbation Method which is appreciate approach for explain and classification of linear and nonlinear equations. This scheme can be vied as an extrusion of a few modern system for determine specified method of nonlinear equations. The numerical decision of using the NHPM to some experiment prove skillfully and reliability of system.
Keywords: HPM, NHPM, Linear and Nonlinear Equation, Burger Equation.

Introduction: The NHPM is capable in observation the comparative or analytic explication of the linear and nonlinear partial differential equation .The suppose decision defend the power, easily as well as simple of the system to implement, in the view we shall illuminate the NHPM represented by [1,2]
The present method is simple, skilful as well as broadly helpful to explain non linear differential equation. In NHPM a homotopy is formulated by suggested and install parameter pϵ(0,1) the HPM use the little parameter as well as the clarification is written as capability range in p in induction of He's polynomials which can be generated by many methods. Particularly physical developments are restrained by linear or non linear differential equation.
Own selves provide the reasoning of the new homotopy perturbation method .personally current numerical decision to determine the ability of the NHPM method for a few PDE .Certainly, provide the conclusions.

Homotopy Perturbation Formula:
We explain this method; let us suppose the following function:
S(u)-t(r) = 0, r $\in \phi$ (1)
With the basic case

$$A\left(u, \frac{\partial u}{\partial n}\right) = 0, r \in \lambda, \qquad (2)$$

Where S is a general operator t(r) is a analytic function, A is a boundary operator, and λ is the boundary of the domain. The operator S can be generally divided into two operator, K and M, Where K is a linear and M a nonlinear operator. Equation (1) can be, therefore,
K(u) +M(u) –t(r) = 0 (3)
Using the homotopy technique, we constructed a homotopy v(r,p): $\phi \times [0,1] \rightarrow$ R, which satisfies

$$H(v,p) = (1-p)[K(v)-K(u_0)] + p[S(v) - t(r)] = 0, \qquad (4)$$

Or

$$H(v,p) = K(v) - K(u_0) + p\,K(u_0) + p\,K(u_0) + p[M(v)-t(r)] = 0, \qquad (5)$$

Where p$\in$[0,1] is called homotopy parameter and u_0 is an initial approximation for the solution of (1), which satisfies the boundary condition. Obviously, from (4) or(5) , we will have

$$H(v,0) = K(v) - K(u_0) = 0 \qquad (6)$$

$$H(v,1) = S(v) - t(r) = 0 \qquad (7)$$

And changing process of p from zero to unity is just that of H(v,p) from K(v)-K(u_0) to S(v)-t(r).

In topology, this is called deformation K(v)-K(u_0) and S(v)-t(r) are called homotopy

We can assume that the solution of (4) or (5) can be expressed as a series in p as follows:

V = v_0 +pv_1 $+ p^2 v_2 + p^3 v_3 + \ldots$ (8)

Setting p=1 result in the approximate solution of (1)

U = $\lim_{p\to 1} v = v_0 + v_1 + v_2 + v_3 + \ldots$ (9)

New Homotopy Perturbation Method:

The general types of PDE can be suppose as the

$\frac{\partial\phi}{\partial s}$ +k(ϕ($q_1, q_2, q_3 \ldots\ldots q_{n-1,}$ s)) = m($q_1, q_2, q_3 \ldots\ldots q_{n-1,}$s) (1)

With the successive basic case :

$\Phi(q_1, q_2, q_3 \ldots\ldots q_{n-1,} s_0)$ =G($q_1, q_2, q_3 \ldots\ldots q_{n-1,}$)

Where k is a non –linear operators which is depends on the function ϕ and its derivatives with respect to q'_j s j = 1 to n-1,s and m is in homogenous for determine equation (1) by applying NHPM we establish the successive homotopy

(1-p)($\frac{\partial\phi}{\partial s}$ - ϕ_0) +p($\frac{\partial\phi}{\partial s} + k(\phi) - m$) = 0 (2)

Or

($\frac{\partial\phi}{\partial s}$) = ϕ_0 -p($\phi_0 + k(\phi) - m$) = 0 (3)

Applying the inverse operator, $L^{-1} = \int_{S_0}^{S}$ ds on two sided of equation (3)

Φ($q_1, q_2, q_3 \ldots\ldots q_{n-1,} s$) = $\Phi(q_1, q_2, q_3 \ldots\ldots q_{n-1,} s_0)$ + $\int_{s_0}^{s} \phi_0$ ds -p$\int_{s_0}^{s}(\phi_0 + k\phi - m)$ ds (4)

Where

$\Phi(q_1, q_2, q_3 \ldots\ldots q_{n-1,} s_0)$ = $\Phi(q_1, q_2, q_3 \ldots\ldots q_{n-1,} s_0)$

Suppose the explanation of equation (4)

$\Phi = \phi_0 + p\phi_1 + p^2\phi_2 + p^3\phi_3 \ldots\ldots\ldots ,\phi_j$ = j= 0,1,2,3…

Are function which should be illuminate consider that the fundamental proximate of the explanation is in the successive form

$\phi_0(q_1, q_2, q_3 \ldots\ldots q_{n-1,}$ s) = $\sum_{j=0}^{\infty} c_j (q_1, q_2, q_3 \ldots\ldots q_{n-1,})\, p_j(s)$

Where

c_j ($q_1, q_2, q_3 \ldots\ldots q_{n-1,}$) are unknown coefficients and $p, p^2, p^3 \ldots\ldots$ are specific function

Comparing the coefficients of p

p^0 : $\phi_0(q_1, q_2, q_3 \ldots\ldots q_{n-1,}$ s) =G($q_1, q_2, q_3 \ldots\ldots q_{n-1,}$)+$\sum_{J=0}^{\infty} c_J \int_{s_0}^{s} p_0(s) ds$

p^1 : $\phi_1(q_1, q_2, q_3 \ldots\ldots q_{n-1,}$ s) +$\sum_{J=0}^{\infty} c_J \int_{s_0}^{s} p_0(s) ds - \int_{s_0}^{s} k(\phi_0) - m) ds$

p^j : $\phi_j(q_1, q_2, q_3 \ldots\ldots q_{n-1,}$s)

= -$\int_{s_0}^{s} k(q_1, q_2, q_3 \ldots\ldots q_{n-1,})$ds

Thus the accurate explanation may be gather as

$\Phi(q_1, q_2, q_3 \ldots\ldots q_{n-1,} s)$ = $\phi_0(q_1, q_2, q_3 \ldots\ldots q_{n-1,}$s) = G($q_1, q_2, q_3 \ldots\ldots q_{n-1,}$)+$\sum_{J=0}^{\infty} c_J \int_{s_0}^{s} p_0(s) ds$

(1). $\frac{\partial\chi}{\partial t}$ +ϕ $\frac{\partial\chi}{\partial\phi}$ +$\psi \frac{\partial\chi}{\partial\psi}$ = 0

Basic conditions

χ(φ,ψ, 0) = φ+ ψ ; χ_t (φ, ψ,0) = 0

The above difficult we write .

χ(φ,ψ, t) = φ+ ψ + $\int_0^t(-\phi \frac{\partial\chi}{\partial\phi}$ -$\psi \frac{\partial\chi}{\partial\psi}$)dt

By the NHPM is

$\chi_0 + p\chi_1 + p^2\chi_2 + p^3\chi_3 + \ldots\ldots\ldots$ = φ+ ψ + $p \int_0^t(-\phi\{(\chi_0)_\phi + p(\chi_1)_\phi + p^2(\chi_2)_\phi$ +….}-(ψ){ $(\chi_0)_\psi + p(\chi_1)_\psi + p^2(\chi_2)_\psi + \ldots$}dt

Equating the range of P .
χ_0= φ+ ψ
χ_1= $\int_0^t - \phi(\chi_0)_\phi$+(-$\psi$)$(\chi_0)_\psi$dt
χ_1 = - (φ+ ψ)t
χ_2= $\int_0^t - \phi(-t)$+(-ψ)$(-t)$dt
χ_2= (φ+ ψ)$\frac{t^2}{2!}$
:
:
:
Continue this process we get

$\chi(\varphi,\psi, t) = \sum_{j=0}^{\infty} \chi_j(\varphi, \psi, t)$

$\chi(\varphi,\psi, t) = (\varphi+\psi) - (\varphi+\psi)t + (\varphi+\psi)\frac{t^2}{2!} + \ldots..$

So the accurate explanation.

$(\varphi+\psi)\, e^{-t}$

(2) Consider the pd equation is

$\frac{\partial\chi}{\partial t} + \phi\, \frac{\partial\chi}{\partial\phi} = 0$

Basic conditions

$\chi(\varphi, 0) = \varphi$; $\chi_t(\varphi,0) = 0$

The above difficult we write.

$\chi(\varphi, t) = \varphi + \int_0^t (-\phi \frac{\partial\chi}{\partial\phi})dt$

By the NHPM is

$\chi_0 + p\chi_1 + p^2\chi_2 + p^3\chi_3 + \ldots\ldots\ldots = \varphi + p\int_0^t (-\phi\{(\chi_0)_\phi + p(\chi_1)_\phi + p^2(\chi_2)_\phi + \ldots.\}dt$

Equating the range of P .

$\chi_0 = \varphi$

$\chi_1 = \int_0^t -\phi(\chi_0)_\phi dt$

$\chi_1 = -\varphi t$

$\chi_2 = \int_0^t -\phi(-t)dt$

$\chi_2 = \varphi\frac{t^2}{2!}$

:

:

:

Continue this process we get

$\chi(\varphi, t) = \sum_{j=0}^{\infty} \chi_j(\varphi, t)$

$\chi(\varphi, t) = \varphi - \varphi t + \varphi\frac{t^2}{2!} + \ldots..$

So the accurate explanation.

$\varphi\, e^{-t}$(3).

Consider the burger equation is

$\chi_t + \chi\chi_\phi - \chi_{\phi\phi} = 0$

$basic$condition

$\chi(\varphi,0) = \varphi$

The above difficult may be write as.

$\chi(\varphi,0) = \varphi + p\int_0^t[-(\chi\,\chi_\phi) + \chi_{\phi\phi}]\,dt$

by the NHPM is

$\chi_0 + p\chi_1 + p^2\chi_2 + p^3\chi_3 + \ldots\ldots\ldots = \varphi + p\int_0^t[-(\chi_0 + p\chi_1 + p^2\chi_2 + p^3\chi_3 + \ldots\ldots\ldots)\{(\chi_0)_\phi + p(\chi_1)_\phi + p2(\chi_2)_\phi + \ldots) + \{(\chi_0)_{\phi\phi} + p(\chi_1)_{\phi\phi} + p^2(\chi_2)_{\phi\phi} + \ldots\}]\,dt$

Equating the range of p.

$\chi_0 = \varphi$

$\chi_1 = \int_0^t[-(\chi_0)(\chi_0)_\phi + (\chi_0)_{\phi\phi}]dt$

$\chi_1 = \int_0^t[-(\phi)(1) + (0)]dt$

$\chi_1 = -t\varphi$

$\chi_2 = \int_0^t[-\{(\chi_0)(\chi_1)_\phi + (\chi_1)(\chi_0)_\phi\} + (\chi_1)_{\phi\phi}]dt$

$\chi_2 = -[(\varphi)(-t) + (-t\varphi)(1) + (0)]$

$\chi_2 = 2\varphi\frac{t^2}{2}$

:

:

:

$\chi(\varphi,t) = \sum_{j=0}^{\infty} \chi_j(\varphi, t)$

$\chi(\varphi,t) = \varphi - t\varphi + \varphi t^2 + \ldots\ldots.$

$\chi(\varphi,t) = \varphi(1 - t + t^2 + \ldots\ldots)$

is become a G.P series so

$\chi(\varphi,t) = \frac{\phi}{1+t}$

thus the accurate explanation. (4).

Consider the burger equation is

$\chi_t - 6\,\chi\chi_\phi - \chi_{\phi\phi} = 0$

$basic$condition

$\chi(\varphi,0) = 6\varphi$

The above difficult may be write as.

$\chi(\varphi,0) = 6\varphi + p\int_0^t[6(\chi\,\chi_\phi) + \chi_{\phi\phi}]\,dt$

by the NHPM is

$\chi_0 + p\chi_1 + p^2\chi_2 + p^3\chi_3 + \ldots\ldots\ldots = 6\varphi + p\int_0^t[6(\chi_0 + p\chi_1 + p^2\chi_2 + p^3\chi_3 + \ldots\ldots\ldots)\{(\chi_0)_\phi + p(\chi_1)_\phi + p2(\chi_2)_\phi + \ldots) + \{(\chi_0)_{\phi\phi} + p(\chi_1)_{\phi\phi} + p^2(\chi_2)_{\phi\phi} + \ldots\}]\,dt$

Equating the range of p.

$\chi_0 = 6\varphi$

$\chi_1 = \int_0^t[6(\chi_0)(\chi_0)_\phi + (\chi_0)_{\phi\phi}]dt$

$\chi_1 = \int_0^t[6(\phi)(6) + (0)]dt$

$\chi_1 = 6^3 t\varphi$

$\chi_2 = \int_0^t[6\{(\chi_0)(\chi_1)_\phi + (\chi_1)(\chi_0)_\phi\} + (\chi_1)_{\phi\phi}]dt$

$\chi_2 = 6[(6\varphi)(-6^3 t) + (-6^3 t\varphi)(6) + (0)]$

$\chi_2 = 26^5\varphi\frac{t^2}{2}$

:

:

:

$\chi(\varphi,t) = \sum_{j=0}^{\infty} \chi_j(\varphi, t)$

$\chi(\varphi,t) = 6\varphi + 6^3 t\varphi + 6^5\varphi t^2 + \ldots\ldots.$

$\chi(\varphi,t) = 6\varphi(1 + 6^2 t + 6^4 t^2 + \ldots\ldots)$

is become a G.P series so

$$\chi(\varphi,t) = \frac{6\phi}{1-36t}$$

thus the accurate explanation.

Conclusion: - In this Paper we used to successfully the NEW Homotopy Perturbation system to get definite explanation for differential equation this system which provide us ideal provocative result in to terms of potential range the NHPS obsolete largely applicative in many fields of science to resolve the particular sample about complication. Furthermore, this method is a powerful tool to solve any different type of PDE. It is also a helpful and useful method to solve the differential equations.

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