MATHEMATICAL MODULATION OF RULE OF MIXTURE FOR THE MULTI-PHASE COMPOSITES

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Abstract

Rule of Mixture (ROM) is one of the methods to find the stiffness of the composite material and its constituents. It is the weighted mean method to evaluate the various properties of the composite especially for continuous fibre and long fibre reinforcement composite [1]. The parameters that matter the Prediction of properties using ROM is the Volume fraction and the young's modulus of the constituents in the composite material. Composite consist of generally two phase one continuous phase is called matrix and the discontinuous phase is termed as reinforcements. Hence, the formulae generated for ROM is generally made for two phase i.e. fibre and matrix. The limitation of the concept is that there is no room for other phases. The following is new concept which is carried out to make it possible to generate multi-phase composite through mathematical modulation of the present Rule of Mixture. Hence, the method is cost effective to find the properties analytically required during a research.

Keywords: Rule of Mixture, Mathematical Modulation, Phases

Introduction: There are several ways to find the properties of the composite one is through experimental through preparing the specimen. Preparing the composite material to test for the application is a costly approach [2]. There will be other factors such as maintaining the matrix to its liquid state and aligning the fibres through it. The next one is the simulation where the software plays as an important part where a composite material is modelled through and finite element analysis done and last and cost effective method of analysis through analytical methods. In those analytical method, Rule of Mixture is one of a kind which provides the prediction of the properties of the composite material. It comes under the Micro mechanical analysis of the composites. In general, the present rule of mixture states about the two phase of the composite. The ROM depends on the Volume fraction and Young's modulus of the constituents.

Objective: Mathematical Modulation of the Rule of mixture. Since it is difficult to evaluate the composite for multi-phase composite. The given equations of rule of mixtures are modulated such that can be incorporated to multiphase. In the present paper we have used just three phases and derived the required equation. The equation derived on are: 1) Axial young's Modulus (E1), 2) Transverse young's Modulus (E2), 3) In-Plane Shear modulus (G12), 4) Major Poisson's ratio and 5) Electrical Conductivity of the composite [3].

Mathematical modulation through derivation of these properties are as follows:

1) **Axial Young's modulus (E1)** :

The total load sustained on the composite $\mathbf{F_c}$ is equal to loads of the constituent's phases of the composite $\mathbf{F_m}, \mathbf{F_f}$ and $\mathbf{F_{nt}}$ (matrix, fibre and CNTs)

$$\mathbf{F_c = F_m + F_f + F_{nt}}$$

From the definition of stress,

$$F = \sigma A$$

Thus the expression becomes,

$$\sigma_c A_c = \sigma_m A_m + \sigma_f A_f + \sigma_t A_{nt}$$

Dividing whole by cross sectional area of the composite, A_c

$$\sigma_c = \sigma_m \cdot \frac{A_m}{A_c} + \sigma_f \cdot \frac{A_f}{A_c} + \sigma_t \cdot \frac{A_{nt}}{A_c}$$

If the composite, matrix and reinforcement phase length are equal, then $\frac{A_m}{A_c}$ is equal to the volume fraction of matrix V_m. Similarly for volume fraction of fibre V_f and volume fraction of CNTs, V_{nt}.

$$\sigma_c = \sigma_m V_m + \sigma_f V_f + \sigma_t V_{nt}$$

For Isostrain state,

$$\epsilon_c = \epsilon_m = \epsilon_f = \epsilon_{nt}$$

When each term divides their respective strain,

$$\frac{\sigma_c}{\epsilon_c} = \frac{\sigma_m}{\epsilon_m}.V_m + \frac{\sigma_f}{\epsilon_f}.V_f + \frac{\sigma_{nt}}{\epsilon_{nt}}.V_{nt}$$

If the deformation of the composite, matrix, fibre and CNTs are elastic then, $\frac{\sigma}{\epsilon}$ = E.

Therefore,

$$E_c = E_m V_m + E_f V_f + E_t V_{nt}$$

Since the composite consist of Matrix, fibre and CNTs that can be written as,

$$V_m + V_f + V_{nt} = 1$$

$$V_m = 1 - V_f - V_{nt}$$

Therefore the equation for three phase of the composite for longitudinal direction can be written as

$$E_c = E_m(1 - V_f - V_{nt}) + E_f V_f + E_{nt} V_{nt}$$

Where, E – Young's modulus

V- Volume fraction

Subscripts m, f and nt represents matrix, fibre and CNTs

2) **Transverse Young's Modulus (E2)**

In this condition, consider the stress in a composite is a same through all phases i.e

$$\sigma_c = \sigma_m = \sigma_f = \sigma_{nt} = \sigma$$

This is Iso-stress condition. The strain/ deformation of the entire composite ϵ_c is

$$\epsilon_c = \epsilon_m V_m + \epsilon_f V_f + \epsilon_{nt} V_{nt}$$

We know that, $\epsilon = \frac{\sigma}{E}$. Therefore,

$$\frac{\sigma_c}{E_c} = \frac{\sigma_m}{E_m}.V_m + \frac{\sigma_f}{E_f}.V_f + \frac{\sigma_{nt}}{E_{nt}}.V_{nt}$$

Since Iso-stress condition

$$\frac{\sigma}{E_c} = \frac{\sigma}{E_m}.V_m + \frac{\sigma}{E_f}.V_f + \frac{\sigma}{E_{nt}}.V_{nt}$$

Here, E_c represents modulus of elasticity in the transverse direction. Dividing whole equation by σ gives,

$$\frac{1}{E_c} = \frac{V_m}{E_m} + \frac{V_f}{E_f} + \frac{V_{nt}}{E_{nt}}$$

This equation can be rewritten as,

$$E_c = \frac{E_m.E_f.E_{nt}}{V_m(E_f.E_{nt}) + V_m(E_f.E_{nt}) + V_m(E_f.E_{nt})}$$

As we know,

$$V_m + V_f + V_{nt} = 1$$

$$V_m = 1 - V_f - V_{nt}$$

Therefore the equation for three phase of the composite for transverse direction can be written as

$$E_2 = \frac{E_m.E_f.E_{nt}}{(1 - V_f - V_{nt})(E_f.E_{nt}) + V_f(E_m.E_{nt}) + V_{nt}(E_f.E_m)}$$

Where, E – Young's modulus

V- Volume fraction

Subscripts m, f and nt represents matrix, fibre and CNTs

3) **Major-Poisson's ratio**

The transverse contraction is denoted as ΔW in the determination of the case along longitudinal direction **E1** which is contributed by all phase i.e. fibre, CNTs and matrix. Thus,

$$\Delta W = \Delta W_f + \Delta W_m + \Delta W_{nt}$$

$$\Delta W = W.V_f v_{12f}\,\epsilon_{11} + W.V_m v_{12m}\,\epsilon_{11} + W.V_{nt} v_{12nt}\,\epsilon_{11} \quad \text{........ (1)}$$

We know that, $v_{12} = -\frac{\epsilon_{22}}{\epsilon_{11}}$

$$\epsilon_{22} = \frac{\Delta W}{W} \quad \text{.................... (2)}$$

Combining (1) and (2)

$$v_{12} = \frac{\Delta W}{W\,\epsilon_{11}}$$

Therefore the equation can be written as

$$v_{12} = V_f v_{12f} + V_m v_{12m} + V_{nt} v_{12nt}$$

4) **In-Plane Shear Modulus**

Consider the shear stress on composite σ_{12}. All the phases i.e. fiber, matrix and Cnts experience same shear stress. Therefore,

$\epsilon_{12m} = \frac{\sigma_{12}}{G_{12m}}$, $\epsilon_{12f} = \frac{\sigma_{12}}{G_{12f}}$ and $\epsilon_{12nt} = \frac{\sigma_{12}}{G_{12nt}}$

Where, G_{12} represents In plane shear modulus

Further, the stain/deformation in the entire composite,

$$\epsilon_c = \epsilon_{12m} V_m + \epsilon_{12f} V_f + \epsilon_{12nt} V_{nt}$$

We know that, $\in_{12m} = \frac{\sigma_{12}}{G_{12}}$

Substituting in the equation we get,

$$\frac{1}{G_{12c}} = \frac{V_m}{G_{12m}} + \frac{V_f}{G_{12f}} + \frac{V_{nt}}{G_{12nt}}$$

Further simplifying we get,

$$\frac{1}{G_{12c}} = \frac{V_m(G_{12f}.G_{12nt})}{G_{12m}} + \frac{V_f(G_{12m}.G_{12nt})}{G_{12f}} + \frac{V_t(G_{12m}.G_{12f})}{G_{12nt}}$$

$$E_2 = \frac{G_{12m}.G_{12f}.G_{12nt}}{V_m(G_{12f}.G_{12nt}) + V_f(G_{12m}.G_{12nt}) + V_t(G_{12m}.G_{12f})}$$

Also, the composite consist of Matrix, fibre and CNTs that can be written as,

$$V_m + V_f + V_{nt} = 1$$

$$V_m = 1 - V_f - V_{nt}$$

Therefore, equation for three phase of the composite for In-plane shear modulus can be written as

$$E_2 = \frac{G_{12m}.G_{12f}.G_{12nt}}{(1 - V_f - V_{nt})(G_{12f}.G_{12nt}) + V_f(G_{12m}.G_{12nt}) + V_{nt}(G_{12m}.G_{12f})}$$

Where, G – shear modulus

V- Volume fraction

5) **Electrical Conductivity**

Conductance of the composite is equal to the sum of the conductance of the three phases. Therefore,

$$C_c = C_m + C_f + C_{nt}$$

We know that conductivity, $\gamma = \frac{C}{A}$

Where, A is the ratio L and a i.e. $A = \frac{a}{L}$

L=length of the composite

a = area of the composite

Implies, $C = \gamma.A$

Substituting in the equation we get,

$$\gamma_c A_c = \gamma_m A_m + \gamma_f A_f + \gamma_t A_{nt}$$

It can be written as,

$$\gamma_c = \gamma_m.\frac{A_m}{A_c} + \gamma_f.\frac{A_f}{A_c} + \gamma_{nt}.\frac{A_{nt}}{A_c}$$

If the composite, matrix and reinforcement phase length are equal, then $\frac{A_m}{A_c}$ is equal to the volume fraction of matrix V_m. Similarly for volume fraction of fibre V_f and volume fraction of CNTs, V_{nt}.

$$\gamma_c = \gamma_m V_m + \gamma_f V_f + \gamma_{nt} V_{nt}$$

Also, the composite consist of Matrix, fibre and CNTs that can be written as,

$$V_m + V_f + V_{nt} = 1$$

$$V_m = 1 - V_f - V_{nt}$$

Therefore the equation for three phase of the composite for electrical conductivity can be written as

$$\gamma_c = \gamma_m (1 - V_f - V_{nt}) + \gamma_f V_f + \gamma_{nt} V_{nt}$$

Where, γ – electrical conductivity

V- Volume fraction

Subscripts m, f and nt represents matrix, fibre and CNTs

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Conclusion: All the above expressions are modulated mathematically form the Rule of Mixture. These are used in the exhibiting future project and comparable results of the composite material. We can derive the equation of these properties and many more by knowing the physic of the equation. It’s a new concept of incorporating three phase using Rules of mixture which is believed to be cost effective.

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