# INTUITIONISTIC FUZZY SEMIPRIME IDEALS IN SEMIGROUPS

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**Abstract**
**In this paper, we introduce the notion of intuitionistic fuzzy semiprimality in a Semigroup which is an extension of fuzzy semiprimality and investigate some of their related properties.**
**Keywords: Intuitionistic fuzzy semigroup, intuitionistic fuzzy ideal, intuitionistic fuzzy bi-ideal, intuitionistic fuzzy semiprime, intuitionistic fuzzy leftsimple**.
**Mathematics Subject Classification (2000): 20M12, 04A72.**

## 1. Introduction

After the introduction of fuzzy sets by L. A. Zadeh[9], the fuzzy set theories have found many applications in the domain of mathematics. The concept of intuitionistic fuzzy sets was introduced by K. T. Atanassov[1, 2], as a generalization of the notion of fuzzy sets.
N. Kuroki[5] introduced and studied fuzzy ideals and fuzzy bi-ideals in semigroups. In this paper, we consider the intuitionistic fuzzification of the concept of a semiprime ideal in a semigroup and some properties of such ideals are investigated.

## 2.Preliminaries

Let S be a semigroup. By a subsemigroup we mean a non-empty subset A of S such that
$A^2 \subseteq A$, and by a left(right) ideal of S we mean a non-empty subset A of S such that $SA \subseteq A$ ($AS \subseteq A$). By two sided ideal or simply ideal, we mean a non-empty subset of S which is both a left and a right ideal of S. A subsemigroup A of a semigroup S is called a bi-ideal of S if $ASA \subseteq A$. A semigroup S is said to be right (resp.left) zero if $xy = y$ ($xy = x$) for all $x, y \in S$. A semigroup S is said to be regular if for each $x \in S$, there exists $y \in S$ such that $x = xyx$. A semigroup S is said to left (resp.right) simple if S itself is the only left (resp.right) ideal of S.
A mapping $\mu : S \rightarrow [0, 1]$ is called a fuzzy set of S and the complement $\bar{\mu}$ is a fuzzy set in S given by $\bar{\mu}(x) = 1 - \mu(x)$ for all $x \in S$.
An intuitionistic fuzzy set (IFS) A in a non empty set X is an object having the form
$A = \{<x, \mu_A(x), \nu_A(x)> / x \in X\}$, where the functions $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ denote the degree of membership and the degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.
**Notation:** For the sake of simplicity, we shall use the symbol $A = <\mu_A, \nu_A>$ for the IFS $A = \{<x, \mu_A(x), \nu_A(x)> / x \in X\}$.

## 3. Intuitionistic fuzzy semiprime ideals

In what follows, let S denote a semigroup unless otherwise specified.
**Definition 3.1.** Let A be an IFS in X and let $t \in [0, 1]$. Then the sets
$U(\mu_A ; t) = \{x \in X : \mu_A(x) \geq t\}$ and $L(\nu_A; t) = \{x \in X : \nu_A(x) \leq t\}$ are called a $\mu$-level t–cut and $\nu$-level t-cut of A, respectively.
**Definition 3.2.** Let A be an intuitionistic fuzzy set of a set X. For each pair $<t, s> \in [0, 1]$, the set $A_{<t, s>} = \{x \in X: \mu_A(x) \geq t \text{ and } \nu_A(x) \leq s\}$ is called the level subset of A.

**Definition 3.3.** An IFS A = <$\mu_A(x)$, $\nu_A(x)$> in S is called an intuitionistic fuzzy subsemigroup of S if

(i) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ii) $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x, y ∈ S.

**Definition 3.4.** An intuitionistic fuzzy Γ-subsemigroup A = <$\mu_A(x)$, $\nu_A(x)$> in S is called an intuitionistic fuzzy left ideal of S if

(i) $\mu_A(xy) \geq \mu_A(y)$,

(ii) $\nu_A(xy) \leq \nu_A(y)$ for all x, y ∈ S.

**Definition 3.5.** An intuitionistic fuzzy subsemigroup A = <$\mu_A(x)$, $\nu_A(x)$> in S is called an intuitionistic fuzzy bi- ideal of S if

(i) $\mu_A(xwy) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ii) $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$ for all w, x, y ∈ S.

**Example3.6.** Let S = {a, b, c, d, e} be a semigroup with the following Cayley table:

| . | a | b | c | d | e |
|---|---|---|---|---|---|
| a | a | a | a | a | a |
| b | a | a | a | a | a |
| c | a | a | c | c | e |
| d | a | a | c | d | e |
| e | a | a | c | c | e |

Clearly S is a semigroup. Define an IFS A = <$\mu_A$, $\nu_A$> in S by $\mu_A(a) = 0.6$, $\mu_A(b) = 0.5$, $\mu_A(c) = 0.4$, $\mu_A(d) = \mu_A(e) = 0.3$ and $\nu_A(a) = \nu_A(b) = 0.3$, $\nu_A(c) = 0.4$, $\nu_A(d) = 0.5$ $\nu_A(d) = 0.6$. By routine calculation we can check that A is an intuitionistic fuzzy bi-ideal of S.

**Definition3.7.** A semigroup S is said to be intuitionistic fuzzy left simple if every intuitionistic fuzzy left ideal of S is constant.

**Theorem 3.8.** If S is left simple, then S is intuitionistic fuzzy left simple.

**Proof.** Let A = <$\mu_A$, $\nu_A$> be an intuitionistic fuzzy left ideal of S and a, b ∈ S.

Since S is left simple, it follows from [3, p.6] that there exist elements x and y in S such that

b = xa and a = yb. then

$\mu_A(a) = \mu_A(yb) \geq \mu_A(b) = \mu_A(xa) = \mu_A(a)$

and $\nu_A(a) = \nu_A(yb) \leq \nu_A(b) = \nu_A(xa) = \nu_A(a)$

So $\mu_A(a) = \mu_A(b)$ and $\nu_A(a) = \nu_A(b)$. Thus A(a) = A(b), which means A is a constant function because a and b are any elements of S. Therefore, S is intuitionistic fuzzy left simple.

**Definition 3.9.** An intuitionistic fuzzy subsemigroup A = <$\mu_A$, $\nu_A$> of S is called an intuitionistic fuzzy interior ideal of S if

(i) $\mu_A(xay) \geq \mu_A(a)$,

(ii) $\nu_A(xay) \leq \nu_A(a)$ for all x, y, a ∈ S.

**Theorem 3.10.** If S is simple, then every intuitionistic fuzzy interior ideal of S is constant.

**Proof.** Let A = <$\mu_A$, $\nu_A$> be an intuitionistic fuzzy left ideal of S and a, b ∈ S. Since S is simple, it follows from [7, Lemma 1.3.9] that there exist elements x and y in S such that a = xby. Since A is an intuitionistic fuzzy interior ideal of S, we have $\mu_A(a) = \mu_A(xby) \geq \mu_A(b)$ and $\nu_A(a) = \nu_A(xby) \leq \nu_A(b)$. It can be seen in a similar way that $\mu_A(b) \geq \mu_A(a)$ and $\nu_A(b) \leq \nu_A(a)$. Since a and b are arbitrary elements, this means that A is a constant function.

**Theorem 3.11.** If S is left simple, then every intuitionistic fuzzy bi-ideal of S is an intuitionistic fuzzy right ideal of S.

**Proof.** Let A = <$\mu_A$, $\nu_A$> be an intuitionistic fuzzy bi- ideal of S and let a, b ∈ S.

Since S is simple, there exists x ∈ S such that b = xa. Since A is an intuitionistic fuzzy bi- ideal of S, it follows that

$\mu_A(ab) = \mu_A(axa) \geq \min\{\mu_A(a), \mu_A(a)\} = \mu_A(a)$ and

$\nu_A(ab) = \nu_A(axa) \leq \max\{\nu_A(a), \nu_A(a)\} = \nu_A(a)$

So A is an intuitionistic fuzzy right ideal.

**Definition3.12.** A subset A of a semigroup S is called semiprime if $a^2 = a$ imply a ∈ A for all a ∈ S.

**Definition3.13.** An IFS A = <$\mu_A$, $\nu_A$> is called intuitionistic fuzzy semiprime if

(i) $\mu_A(x) \geq \mu_A(x^2)$,

(ii) $\nu_A(x) \leq \nu_A(x^2)$ for all x ∈ S.

**Example3.14.** Let S = {o, e, f, a, b} be a set with the following Cayley table:

| . | o | e | f | a | b |
|---|---|---|---|---|---|
| o | o | o | o | o | o |
| e | o | e | o | a | o |
| f | o | o | f | o | b |
| a | o | a | o | o | e |
| b | o | o | b | f | o |

Clearly S is a semigroup. Define an IFS A = <$\mu_A$ , $\nu_A$> in S by $\mu_A(e) = \mu_A(f) = 1$, $\mu_A(a) = \mu_A(b) = \mu_A(o) = 0$ and $\nu_A(e) = \nu_A(f) = 0$, $\nu_A(a) = \nu_A(b) = \nu_A(o) = 1$. By routine calculation A is an intuitionistic fuzzy semiprime of S.

**Theorem 3.15.** If A is semiprime, then $(\chi_A, \overline{\chi}_A)$ is intuitionistic fuzzy semiprime.

**Proof.** Let $a \in S$. If $a^2 \in A$, then since A is semiprime, we have $a \in A$. Thus $\chi_A(a) = 1 \geq \chi_A(a^2)$ and $\overline{\chi}_A(a) = 1 - \chi_A(a) \leq 1 - \chi_A(a^2) = \overline{\chi}_A(a^2)$. If $a^2 \notin A$, then we have $\chi_A(a^2) = 0$. Therefore $\chi_A(a) \geq 0 = \chi_A(a^2)$ and $\overline{\chi}_A(a^2) = 1 - \chi_A(a^2) \geq 1 - \chi_A(a) = \overline{\chi}_A(a)$.

This proves the theorem.

**Theorem 3.16.** Let A be a non-empty subset of S. If $(\chi_A, \overline{\chi}_A)$ is intuitionistic fuzzy semiprime**,** then A is semiprime.

**Proof.** Let $(\chi_A, \overline{\chi}_A)$ be intuitionistic fuzzy semiprime and let $a^2 \in A$ Then $\chi_A(a) \geq \chi_A(a^2) = 1$. So $a \in A$, $\overline{\chi}_A(a) \leq \overline{\chi}_A(a^2) = 1 - \chi_A(a^2) = 1 - 1 = 0$. That is, $\chi_A(a) = 1$. This shows that $a \in A$.

This proves the theorem.

**Theorem 3.17.** For any intuitionistic fuzzy subsemigroup $A = <\mu_A, \nu_A>$ of S, if A is intuitionistic fuzzy semiprime, then $A(a) = A(a^2)$.

**Proof.** Since A is intuitionistic fuzzy semiprime of S, we have

$$\mu_A(a) \geq \mu_A(a^2) = \min\{\mu_A(a), \mu_A(a)\} = \mu_A(a)$$

$$\nu_A(a) \geq \nu_A(a^2) = \max\{\nu_A(a), \nu_A(a)\} = \nu_A(a)$$

That is, $\mu_A(a) = \mu_A(a^2)$ and $\nu_A(a) = \nu_A(a^2)$

This proves the theorem.

**Definition 3.18.** A semigroup S is called left (resp.right) regular if, for each element a of S, there exists an element x in S such that $a = xa^2$ $(a = a^2x)$.

**Theorem3.19.** Let S be left regular. Then, every intuitionistic fuzzy left ideal of S is intuitionistic fuzzy semiprime.

**Proof.** Let $A = <\mu_A, \nu_A>$ of be an intuitionistic fuzzy left ideal of S and let $a \in S$. Then, there exists an element x in S such that $a = xa^2$ since S is left regular. So, we have $\mu_A(a) = \mu_A(xa^2) \geq \mu_A(a^2)$ and $\nu_A(a) = \nu_A(xa^2) \leq \nu_A(a^2)$.

This proves the theorem.

**Definition3.20.** A semigroup S is called intra-regular if, for each element a of S, there exist elements x and y in S such that $a = xa^2y$.

**Theorem3.21.** Let $A = <\mu_A, \nu_A>$ be an intuitionistic fuzzy ideal of S. If S is intra-regular, then A is intuitionistic fuzzy semiprime.

**Proof.** Let a be any element of S. Then, since S is intra regular, there exist x and y in S such that $a = xa^2y$. So, we have $\mu_A(a) = \mu_A(xa^2y) \geq \mu_A(a^2y) \geq \mu_A(a^2)$ and $\nu_A(a) = \nu_A(xa^2y) \leq \nu_A(a^2y) \leq \nu_A(a^2)$

This proves the theorem.

**Theorem3.22.** Let $A = <\mu_A, \nu_A>$ be an intuitionistic fuzzy interior ideal of S. If S is Intra- regular, then A is intuitionistic fuzzy semiprime.

**Proof.** Let $a \in S$. Then since S is intra-regular, there exist x and y in S such that $a = xa^2y$, So we have $\mu_A(a) = \mu_A(xa^2y) \geq \mu_A(a^2)$ and $\nu_A(a) = \nu_A(xa^2y) \leq \nu_A(a^2)$.

This proves the theorem.

**Definition 3.23.** A semigroup S is called Archimedean if, for any elements a, b, there exists a
positive integer n such that $a^n \in SbS$.

**Theorem3.24.** Let S be an Archimedean semigroup. Then, every intuitionistic fuzzy semiprime fuzzy ideal of S is a constant function.

**Proof.** Let $A = <\mu_A, \nu_A>$ be any intuitionistic fuzzy semiprime ideal of S and $a, b \in S$ .
Then since S is Archimedean, there exist x and y in S such that $a^n = xby$ for some integer n. Then, we have $\mu_A(a) = \mu_A(a^n) = \mu_A(xby) \geq \mu_A(b)$ and

$$\mu_A(b) = \mu_A(b^n) = \mu_A(xay) \geq \mu_A(a)$$

Thus we obtain $\mu_A(a) = \mu_A(b)$

Also, we have $\nu_A(a) = \nu_A(a^n) = \nu_A(xby) \leq \nu_A(b)$ and

$$\nu_A(b) = \nu_A(b^n) = \nu_A(xay) \leq \nu_A(a)$$

Therefore we have $A(a) = A(b)$ for all $a, b \in S$.

This proves the theorem.

## REFERENCES


[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems 20(1) (1986), 87-96.

[2] K. T. Atanassov, New operations defined over intuitionistic fuzzy sets, Fuzzy sets and Systems 61(1994), 137-142.

[3] A. H. Clifford and G. B. Preston, 1961, the algebraic theory of semigroups, American Math. Soc., providence, RI,I.

[4] K. H. Kim, On intuitionistic Q-fuzzy Semiprime ideals in Semigroups, Advances in fuzzy mathematics,Vol.1(2006)15-21.

[5] N. Kuroki, On fuzzy ideals and fuzzy-bi-ideals in Semigroups, Fuzzy sets and systems 5(1981) 203-215.

[6] N. Kuroki. Fuzzy semiprime ideals in Semigroups, Fuzzy sets and systems 8(1982)71-79.

[7] M.Petrich, 1973, Introduction to semigroups, Merril, Columbus, OH.

[8] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35(1971) 512-517.

[9] L. A. Zadeh, Fuzzy sets, Information and Control**,** 8 (1965) 338-353.