KINEMATIC DESIGN OF PARALLEL MANIPULATOR USING ISOTROPIC INDICES

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Abstract
The kinetostatic performance of parallel manipulator depends on the kinematic structure and the manipulator posture of a particular kinematic structure in the work space. The manipulator performance is analysed by condition number which depends on the manipulator jacobian. This paper investigates the different isotropic postures which define isotropic index based on velocity. In this proposal, the concept of isotropy is used for the kinematic design of parallel manipulators.

Keywords: Parallel Manipulators, Condition Number, Velocity Isotropy.

1 INTRODUCTION
A robot is known as Isotropic, if kinetostatic properties are homogeneous with respect to all the directions within the confined work space (1). Parallel Manipulators have a kinematic structure that has some more benefits over serial manipulator. Accuracy, high speed and high load to weight ratio are the benefits of parallel manipulator. The idea of robot isotropy was firstly initiated by Salisbury and Craig in 1982 (2). Since 1982, a lot of research gone through for parallel manipulators, particularly for design and controlling purpose. Salisbury and Craig proposed a new performance index called condition number of the Jacobian matrix. When the condition number of this matrix reaches the least value of unity, the manipulator is called Isotropic (3). Various performance indices are evaluated to attain accuracy of manipulator structures was proposed by Stephen L.chiu (4). This theory discuss about the condition number, though it is not a true measure of optimum. It is related to manipulability in inverse proportion. As we know the main measure of the manipulability is the transmission ratio. To evaluate this transmission ratio, the concept of velocity ellipsoid came into picture. It helps us to develop the relation between transmission ratios and manipulator performance. The planar manipulator of two, three and four degrees of freedom and 3 DOF spatial manipulator contains few isotropic postures which are determined by Kircanski (5). Gosselin and Angeles (6) proposed a new concept called Global condition index. It defines the optimized kinematic analysis of Robot manipulator. In the entire workspace the GCI helps us to estimate the distribution of condition number. The dexterity indices that are used to define the performance characteristics of planar and spatial manipulators are proposed by Gosselin (7) based on the condition number of the Jacobian matrix.

CONDITION NUMBER
The condition number of Jacobian matrix
\[ k = \|J\|\|J^{-1}\| \]

The condition number defines an upper bound for the amplification of the relative error. This definition of the condition number can be used with different matrix norms. In this paper, the Euclidean – or Frobenius – norm was used which is defined as:
\[ ||J|| = \sqrt{\text{tr}(J^TWJ^T)} \]

Where \( W \) is the weight matrix

\[ W = \frac{1}{n} \]

and \( J \) is assumed to be \( n \times n \) or equivalently

\[ ||J|| = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}|^2} \]

### VELOCITY ISOTROPY

A manipulator, if it can perform the same velocity in all the directions then it is said to be isotropic with respect to the velocity. It is evaluated by means of velocity ellipsoid.

### II INVERSE KINEMATIC ANALYSIS OF PARALLEL MANIPULATOR

![Fig.1 Kinematic model of 3RPR Manipulator](image)

The dimension of the manipulator is denoted by \( L \). \( R_i \) is the length of the \( i \)th actuator, where \( i = 1,..3 \) and by \( O(x,y) \) the coordinates of the point of attachment of the \( i \)th leg to the moving platform. Moreover, the position of the point of the attachment of the \( i \)th leg to the base is given by \((x_{oi}, y_{oi})\). As in the case of the manipulator with revolute actuators, the Cartesian coordinates are given by the position of the centroid of the moving platform \( O(x,y) \) and by its orientation, defined here by angle \( \Psi \).

We can write as

\[ x_i = x - L \cos \Psi_i - x_{oi}, \quad i = 1,2,3 \]

\[ y_i = y - L \sin \Psi_i - y_{oi}, \quad i = 1,2,3 \]

where angles \( \Psi_i \) and the pairs \((x_{oi}, y_{oi})\) are given by

\[ \Psi_1 = \Psi + \pi/6 \]

\[ \Psi_2 = \Psi + 5\pi/6 \]

\[ \Psi_3 = \Psi - \pi/2(2) \]

\[ \{x_{01},x_{02},x_{03}\} = \{0,1,1/2\} \] (3)

\[ \{y_{01},y_{02},y_{03}\} = \{0,0,\sqrt{3}/2\} \] (4)

The inverse kinematic problem, which has only one solution here, it can be solved using:

\[ R_i = \sqrt{x_i^2 + y_i^2}, \quad i = 1,2,3 \] (5)

Therefore, given a certain position and orientation of the manipulator, the required length of the actuators can be computed directly from equation (5).

### III JACOBIAN OF A PARALLEL MANIPULATOR

The Jacobian matrix of the parallel manipulator with prismatic actuators is defined similarly to the one of the manipulator with revolute actuators given as

\[ \hat{J}e = \hat{T} \]

Where \( e = [x,y,\Psi]^T \) is the vector of Cartesian velocities and \( \hat{T} = [T_1,T_2,T_3]^T \) is the vector of linear actuator rates.

The differentiation \( R_i \) of \( \hat{T} \) leads to the Jacobian matrix

Where

\[ p_i = x - x_{oi} - L\cos \Psi_i \] (7)

\[ q_i = y - y_{oi} - L\sin \Psi_i \] (8)

\[ s_i = (x - x_{oi})L\sin \Psi_i - (y - y_{oi})L\cos \Psi_i \]

And the angles \( \Psi_i \), for \( i = 1,2 \) and 3 are defined as in equation(2).

### IV RESULTS

![Fig.2 Condition Number vs Vertical Reach](image)
CONCLUSION

This paper discusses about the condition number of 3 RPR parallel manipulator. The results are plotted against the vertical reach of the manipulator. We confined only an intermediate range for the vertical reach in the workspace. According to the two best postures, the condition number is plotted as the least value at a particular range of vertical reach. That region is considered as nearly isotropic region. The best isotropic point is plotted at the lowest point of the condition number. The highest peak of the condition number is considered as near singular region. So the operation of the 3 RPR manipulator is designed in the isotropic region of the entire workspace.

REFERENCES