THE STUDY OF CARTESIAN PRODUCT BASED ON THE THEORY OF BAGS AND FUZZY BAGS

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Abstract

In this paper, we seek the existence of some relations between two bags as well as two fuzzy bags. For that we need the notion of Cartesian product of two bags and two fuzzy bags which have been defined and established few theorems.

Key words: algebra, bags, fuzzy bags, Cartesian product.

1. Introduction:-

The theory of bags as a natural extension of set theory was introduced by cerfetal in 1971, Peterson in 1976 and Yager in 1986. Many result have been established by these authers. Bags have been observed to be an important concept in many information processing, like SELECT in rational data base. The concept of bags with fuzzy elements, which called as fuzzy bags introduced by [1], and was further studied by [2]. In case of a fuzzy bag, an element may apper repeatedly with different membership grades. Further some operations on bags and fuzzy bags such as sum, removal, union, intersection introduced by [3] and [4].

In this section we study some relations between two bags and two fuzzy bags as such we need the Cartesian product of bags and fuzzy bags have been defined and established some theorems.

2.1 Definitions of Bags:-

A bag B drawn from a set 'X' is represented by a function count B or C_B defined as:-

$$C_B: X \rightarrow N$$

Where N represents the set of non negative integers.

A bag B is a set if $C_B(X) = 0 \; or \; 1 \; for \; all \; x \in X$

Some other definitions are as follows:-

2.1.1 The support set

$$B^* = \{x \in X: C_B(x) > 0\}$$

2.1.2 If B_1 and B_2 are two bags drawn from a set X, we call B_1 to be a sub bag of B_2 denoted by $B_1 \subseteq B_2 \; if \; for \; all \; x \in X,$

$$C_{B_1}(x) \leq C_{B_2}(x)$$

2.1.3 Two bags B_1 and B_2 are said to be equal if $B_1 \subseteq B_2 \; and \; B_2 \subseteq B_1$ denoted by $B_1=B_2$.

2.1.4 B_1 is said to be proper sub bag of B_2 denoted by $B_1 \subset B_2, if \; for \; every$ $x \in X, C_{B_1}(x) \leq C_{B_2}(x)$ and there exists at least one $x \in X \; such \; that$ $C_{B_1}(x) < C_{B_2}(x)$

2.1.5 A bag B is said to be empty if $C_B(x) = 0, for \; all \; x \in X.$

2.1.6 A cardinality of a bag B drawn from a set X is denoted by card (B) and is given by

$$\text{Card}(B) = [B] = \sum_{x \in X} C_B(x)$$

2.1.7 The insertion of x into a bag B results in a new bag B denoted by $B \oplus x$ such that

$C_B(x)= C_B(x)+1$

$C_B(y)= C_B(y)$, for all $y \neq x$.

2.1.8 Addition of two bags B_1 and B_2 drawn from a set X results in a new bag i.e B= $B_1 \oplus B_2$ such that for all $x \in X$,

$$C_B(x) = C_{B_1}(x) + C_{B_2}(x)$$

2.1.9 The removal of x from the bag B results in a new bag B' denoted by

$$B' = B \ominus x \; such \; that$$

$$C_{B'}(x) = \max\{C_B(x) - 1, 0\}$$

$$C_{B'}(y) = C_B(y), for \; all \; y \neq x$$

2.1.10 If B_1 and B_2 are two bags drawn from a set X, the removal of the bag B_2 from the bag B_1 results in a bag B, denoted by $B = B_1 \ominus B_2$ Such that for all $x \in X$,

$C_B(x) = \max\{C_{B_1}(x) - C_{B_2}(x), 0\}$

2.1.11 The union of two bags B_1 and B_2 drawn from a set X is a bag B denoted by $B = B_1 \cup B_2$ such that for all $x \in X$, $C_B(x) = \max\{C_{B_1}(x), C_{B_2}(x)\}$

2.1.12 The intersection of two bags B_1 and B_2 drawn from a set X is a bag B denoted by $B = B_1 \cap B_2$ such that for all $x \in X$

$$C_B(x) = \min\{C_{B_1}(x), C_{B_2}(x)\}$$

2.1.13 Two bags A and B are said to be equivalent if and only if $|A| = |B$

2.2 Results on Bags

Let A, B, C are three bags drawn from the same domain X, then we have as follows:-

I. $A \cup B = B \cup A$

II. $A \cap B = B \cap A$

III. $A \oplus B = B \oplus A$

IV. $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

V. $A \cap B \subseteq A \subseteq A \cup B \subseteq A \oplus B$

VI. $A \ominus B \subseteq A \subseteq A \oplus B$

2.3 Cartesian product of Bags:-

Definition 2.3.1:-

Let A and B are two bags drawn from the domain X. Then their Cartesian product denoted by $A \otimes B$ is a bag drawn from the domain $X \times Y$ such that for every

$(x, y) \in X \times Y$, we have

$$C_{A\otimes B}(x, y) = C_A(x). C_B(y)$$

Note 2.3.1:-

The two bags can be drawn from the same domain.

Example 2.3.1:-

Let A=$\{\frac{x_1}{2}, \frac{x_2}{3}, \frac{x_3}{1}\}$

And B==$\{\frac{y_1}{2}, \frac{y_2}{3}, \frac{y_3}{3}\}$ are two bags drawn from the domain

X={x_1, x_2, x_3, y_1, y_2, y_3}, then

$$A \otimes B = \{\frac{(x_1, y_1)}{2}, \frac{(x_1, y_2)}{6}, \frac{(x_1, y_3)}{6}, \frac{(x_2, y_1)}{3}, \frac{(x_2, y_2)}{9}, \frac{(x_2, y_3)}{9}, \frac{(x_3, y_1)}{1}, \frac{(x_3, y_2)}{3}, \frac{(x_3, y_3)}{3}\}$$

Note 2.3.2 :-

If A and B are two bags drawn from the domain X=$\{x_1, x_2, x_3,\}$, then

$C_{A\otimes B}(x_1, x_2) \neq C_{A\otimes B}(x_2, x_1)$,in general

Example 2.3.2:-

Let A=$\{\frac{x_1}{2}, \frac{x_2}{4}, \frac{x_3}{1}\}$

And B=$\{\frac{x_1}{1}, \frac{x_2}{3}, \frac{x_3}{5}\}$ are two bags drawn from the domain

X={x_1, x_2, x_3}, then

$$C_{A\otimes B}(x_1, x_2) = 6$$

And $$C_{A\otimes B}(x_2, x_1) = 4$$

Hence $C_{A\otimes B}(x_1, x_2) \neq C_{A\otimes B}(x_2, x_1)$

Note 2.3.3

If A and B are two bags drawn from the domain X={x_1, x_2, x_3} then

$C_{A\otimes B}(x_1, x_2) \neq C_{B\otimes A}(x_1, x_2)$ in general

Theorem 2.3.1:-

For any two bags A and B drawn from the domain X, then

i. $A \otimes B \neq B \otimes A$, in general.

ii. For all x, y ∈ X, then

$$C_{A\otimes B}(x, y) = C_{B\otimes A}(y, x)$$

From example (2.3.1), it can be easily verified.

Theorem 2.3.2:-

For any three bags A, B, C drawn from the domain X, then

i. $A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$

ii. $A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$

Proof

i. For all x ,y ∈ X ,then we have

$C_{A\otimes(B\cup C)}(x, y)$

$= C_A(x). C_{(B\cup C)}(y)$

$=C_A(x). \{\max(C_B(y), C_C(y))\}$

$=\max\{C_A(x). C_B(y). C_A(x). C_C(y)\}$

$=\max\{C_{A\otimes B}(x, y), C_{A\otimes C}(x, y)\}$

$=C_{(A\otimes B)\cup(A\otimes C)}(x, y)$

Thus established the theorem. Similarly the proof of (ii) can be established.

Theorem 2.3.3:-

For any three bags A, B, C drawn from the domain X, then

i. $A \otimes (B \oplus C) = (A \otimes B) \oplus (A \otimes C)$

ii. $A \otimes (B \ominus C) = (A \otimes B) \ominus (A \otimes C)$

Proof

i. Let for all x, y ∈ X, such that

$C_{A\otimes(B\oplus C)}(x, y) = C_A(x). C_{(B\oplus C)}(y)$

$= C_A(x). [C_B y + C_c(y)]$

$=C_A(x). C_B(y) + C_A(x). C_C(y)$

$=C_{A\otimes B}(x, y) + C_{A\otimes C}(x, y)$

$=C_{(A\otimes B)\oplus(A\otimes C)}(x, y)$

This proofs the theorem.

ii. Again, let for all x, y∈ X, so that

$$C_{A\otimes(B\ominus C)}(x,y) = C_A(x).C_{(B\ominus C)}(y)$$
$$= c_A(x).\{\max(C_B(y) - C_C(y),0)\}$$
$$= \max\{(c_A(x).C_B(y) - c_A(x).C_C(y),0)\}$$
$$= \max\{(C_{A\otimes B}(x,y) - C_{A\otimes C}(x,y),0)\}$$
$$= C_{(A\otimes B)\ominus(A\otimes C)}(x,y)$$

Which completes the proof.

3.1 Definitions of Fuzzy bags :-

A fuzzy bag 'F' drawn from a set 'X' is characterized by a count functions,

$$CF: X \times I \to N,$$

Where I is the unit interval [0,1] and N is the set of non-negative integers.

There are some other definitions as follows:-

3.1.1:- Two fuzzy bags F_1 and F_2 drawn from a set X are said to be equal denoted by $F_1 = F_2$ if for all

x ∈ X and $\alpha \in I$, then $CF_1(x,\alpha) = CF_2(x,\alpha)$.

3.1.2:- For any two fuzzy bags F_1 and F_2 drawn from a set X, F_1 is said to be a fuzzy sub bag of F_2 denoted by $F_1 \subseteq F_2$, if for all x ∈ X and $\alpha \in I$, then $CF_1(x,\alpha) \leq CF_2(x,\alpha)$

3.1.3:- Let F be a fuzzy bag drawn from a set X, then the fuzzy supporting set of F is a fuzzy sub set of X, whose membership function is given by $F^*(x) = \max\{\alpha: CF(x,\alpha) > 0\}$,

$if\{\alpha, CF(x,\alpha) > 0\}$

Is non empty

0, if CF(x, α)=0, for all α

3.1.4:-The sum of two fuzzy bags F_1 and F_2 drawn from a set X results in a fuzzy bag F denoted by $F = F_1 \oplus F_2$ such that for all x ∈ X and $\alpha \in I$, then

CF(x, α)= CF_1(x, α)+ CF_2(x, α).

3.1.5:- Let F_1 and F_2 are two fuzzy bags drawn from a set X. The removal of the fuzzy bag F_2 from the fuzzy F_1 results in a fuzzy bag F denoted by

$F = F_1 \ominus F_2$ such that for all x ∈ X and $\alpha \in I$, then

CF(x, α) = max{(CF_1(x, α)- CF_2(x, α)),0}.

3.1.6:- The union F of two fuzzy bags F_1 and F_2 drawn from a set X is denoted by $F = F_1 \cup F_2$ such that for all x ∈ X and $\alpha \in I$,

CF(x, α)= max{(CF_1(x, α), CF_2(x, α)}.

3.1.7:- The intersection F of two fuzzy bags F_1 and F_2 drawn from a set X is denoted by $F = F_1 \cup F_2$ such that for all x ∈ X and $\alpha \in I$,

CF(x, α) = min{(CF_1(x, α), CF_2(x, α)}.

3.1.8:- Let F be a fuzzy bag drawn from a set X, then the insertion of an element y with membership value 'a' into F results a new fuzzy bag F_1 such that

$$CF_1(y,\alpha) = CF(y,\alpha) + 1, \alpha = a$$
$$CF_1(y,\alpha) = CF(y,\alpha), \alpha \neq a$$
$$CF_1(x,\alpha) = CF(x,\alpha), x \neq y$$

3.1.9:- Let F be a fuzzy drawn from a set X. Then the removal of an element y with membership value 'a' from F results in a new fuzzy bag

$F_1 = F \ominus y$ Such that

$$CF_1(y,\alpha) = \max\{CF(y,\alpha) - 1,0\}, \alpha = a$$
$$CF_1(y,\alpha) = CF(y,\alpha), \alpha \neq a$$
$$CF_1(x,\alpha) = CF(x,\alpha), x \neq y$$

3.1.10:- If F be a fuzzy bag drawn from a finite set X, then the cardinality of F is denoted by $|F|$ is given by $|F| = \sum_{x\in X} \sum_{0<\alpha\leq 1} CF(x,\alpha)$

When ever it exists.

3.2:- Properties of fuzzy bags:-

Let F_1 , F_2 and F_3 are fuzzy bags drawn from a set X, then

I. $F_1 \cup F_2 = F_2 \cup F_1$
II. $F_1 \cap F_2 = F_2 \cap F_1$
III. $F_1 \oplus F_2 = F_2 \oplus F_1$
IV. $(F_1 \cup F_2) \cup F_3 = F_1 \cup (F_2 \cup F_3)$
V. $(F_1 \cap F_2) \cap F_3 = F_1 \cap (F_2 \cap F_3)$
VI. $(F_1 \oplus F_2) \oplus F_3 = F_1 \oplus (F_2 \oplus F_3)$
VII. $F_1 \cap F_2 \subseteq F_1 \subseteq F_1 \cup F_2 \subseteq F_1 \oplus F_2$
VIII. $F_1 \ominus F_2 \subseteq F_1 \subseteq F_1 \oplus F_2$
IX. $|F_1 \cup F_2| \leq |F_1| + |F_2|$
X. $|F_1 \oplus F_2| = |F_1| + |F_2|$

(This can be extended for finite number
Of fuzzy bags)

XI. $|F_1 \cap F_2| \geq |F_1| \ominus |F_2|$

3.3:- Cartesian product of fuzzy bags:-

Definition 3.3.1:-

Consider two finite fuzzy bags F_1 and F_2 drawn from a set X, then their Cartesian product denoted by $F_1 \otimes F_2$ is a fuzzy bag drawn from $X \times X$, such that for all (x, y) ∈ $X \times X$ so that $C(F_1 \otimes F_2)((x,y),\gamma)$ = CF_1(x, α). CF_2(y, β)

Where $x, y \in X$ and $\alpha, \beta, \gamma \in I$ $with$ $\gamma = \alpha.\beta$

Note:3.3.1:-

Two finite fuzzy bags can also be drawn from the same set.

Example:3.3.1:-

Let x={a, b} and y={m, n}∈X then

$$F_1 = \{(a,.2)/2, (a,.5)/4, (b,.5)/1\}$$

And

$F_2 = \{(m,.3)/4, (n,.5)/3, (n,.6)/5\}$, then

$$F_1 \otimes F_2 = \{\frac{((a,m),.06)}{8}, \frac{((a,m),.15)}{16}, \frac{((a,n),.1)}{6}, \frac{((a,n),.12)}{10}, \frac{((a,n),.25)}{12}, \frac{((a,n),.3)}{20}, \frac{((b,m),.15)}{4}, \frac{((b,n),.12)}{3}, \frac{((b,n),.3)}{5}\}$$

Note 3.3.2:-

For any two fuzzy bags F_1 and F_2 drawn from the set X, then

i. $F_1 \otimes F_2 \neq F_2 \otimes F_1$, in general.

ii. For all x, y $\in$ X and $\gamma = \alpha.\beta, \alpha, \beta, \gamma \in I$

Then $C(F_1 \otimes F_2)((x,y),\gamma) = C(F_2 \otimes F_1)((y,x),\gamma)$

The above note (3.3.2) can be easily verified by using the example (3.3.1).

Theorem 3.3.1:-

For any three finite fuzzy bags A, B and c drawn from the set X, then

i. $A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$

ii. $A \otimes (B \cap C) = (A \otimes B) \bigcap (A \otimes C)$

Proof

(i)For any $x, y \in X$ and $\alpha, \beta, \gamma \in I$ $with$ $\alpha.\beta = \gamma$, then

C $\{A \otimes B \cup A \otimes C\}((x,y),\gamma)$

= max $\{C(A \otimes B)((x,y),\gamma), C(A \otimes C)((x,y),\gamma))\}$

= max $\{CA(x,\alpha).CB(y,\beta), CA(x,\alpha).CC(y,\beta)\}$

$= CA(x,\alpha).\max\{CB(y,\beta), CC(y,\beta)\}$

$= CA(x,\alpha).C(B \cup C)(y,\beta)$

= C $(A \otimes (B \cup C))((x,y),\gamma), \gamma = \alpha.\beta$

This completes the proof of (i) and proof of (ii) is similar

Example 3.3.2:-

Let X= {a, b} be a set, then

A= {(a, .2)/2, (a, .5)/4, (b,.5)/1}

B= {(a, .3)/4, (b, .6)/5} and

C= {(a, .5)/2, (b, .7)/3, (b, .4)/6} so

We have

$B \cup C =$ {(a, .3)/4, (a, .5)/2, (b, .4)/6, (b, .6)/5, (b, .7)/3}

$$A \otimes (B \cup C) = \{\frac{((a,a),.06)}{8}, \frac{((a,a),.1)}{4}, \frac{((a,a),.15)}{16}, \frac{((a,a),.25)}{8}, \frac{((a,b),.08)}{12}, \frac{((a,b),.12)}{10}, \frac{((a,b),.14)}{6}, \frac{((a,b),.2)}{24}, \frac{((a,b),.3)}{20}, \frac{((a,b),.35)}{12}, \frac{((b,b),.2)}{6}, \frac{((b,b),.3)}{5}, \frac{((b,b),.35)}{3}, \frac{((b,a),.15)}{4}, \frac{((b,a),.25)}{2}\}$$

$$A \otimes B = \{\frac{((a,a),.06)}{8}, \frac{((a,a),.15)}{16}, \frac{((a,b),.12)}{10}, \frac{((a,b),.3)}{20}, \frac{((b,b),.3)}{5}, \frac{((b,a),.15)}{4}\}$$

$$A \otimes C = \{\frac{((a,a),.1)}{4}, \frac{((a,a),.25)}{8}, \frac{((a,b),.14)}{6}, \frac{((a,b),.08)}{12}, \frac{((a,b),.35)}{12}, \frac{((a,b),.2)}{24}, \frac{((b,b),.35)}{3}, \frac{((b,b),.2)}{6}, \frac{((b,a),.25)}{2})\}$$

Now, $(A \otimes B) \cup (A \otimes C)$

$$= \{\frac{((a,a),.06)}{8}, \frac{((a,a),.1)}{4}, \frac{((a,a),.15)}{16}, \frac{((a,a),.25)}{8}, \frac{((a,b),.12)}{10}, \frac{((a,b),.3)}{20}, \frac{((a,b),.14)}{6}, \frac{((a,b),.08)}{12}, \frac{((a,b),.35)}{12}, \frac{((a,b),.2)}{24}, \frac{((b,b),.3)}{5}, \frac{((b,b),.35)}{3}, \frac{((b,b),.2)}{6}, \frac{((b,a),.15)}{4}, \frac{((b,a),.25)}{2}\}$$

Hence $A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$

Theorem 3.3.2:-For any three finite fuzzy bags A, B, and C drawn from the set X, then

i. $A \otimes (B \oplus C) = (A \otimes B) \oplus (A \otimes C)$

ii. $A \otimes (B \ominus C) = (A \otimes B) \ominus (A \otimes C)$

Proof(i)

For any $x, y \in X$ and $\alpha, \beta, \gamma \in I$ $with$ $\gamma = \alpha.\beta$

$C(A \otimes (B \oplus C))((x,y),\gamma)$

$= CA(x,\alpha).\,C(B\oplus C)(y,\beta)$

$= CA(x,\alpha).\,\{CB(y,\beta) + CC(y,\beta)\}$

$= CA(x,\alpha).\,CB(y,\beta) + CA(x,\alpha).\,CC(y,\beta)$

$= C(A\otimes B)\big((x,y),\gamma\big) + C(A\otimes C)\big((x,y),\gamma\big)$

$= C[(A\otimes B)\oplus(A\otimes C)],\big((x,y),\gamma\big)$

This establishes (i) and proof of (ii) is similar.

Conclusion:-

In this paper, we have studied some definitions, results and properties of bags and fuzzy bags. In this connection we have established the relations between two bags as such as two fuzzy bags, so we need the Cartesian product of both bags and fuzzy bags defined and some theorems also established on bags and fuzzy bags.

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