ANALYSIS OF MIMO-OFDM UNDER RAYLEIGH FADING IN 4G CELLULAR SYSTEMS

Prof. M. Sushanth Babu1, K. Ragini2
1 Professor in Department of Electronics & Communication Engineering, Vardhaman College of Engineering, Shamshabad, Hyderabad.
2 Assistant Professor in Department of Electronics & Communication Engineering, Mallareddy Engineering College for Women, Dhillapally, Hyderabad.
Email: sushanth19.m@gmail.com1, raginikotagiri@gmail.com2

Abstract
MIMO-OFDM technology is a combination of multiple-input multiple-output (MIMO) wireless technology with orthogonal frequency division multiplexing (OFDM) that has been recognized as one of the most promising techniques to support high data rate and high performance in different channel conditions. Again, Alamouti’s space time block coding scheme for MIMO system has drawn much attention in 4G wireless technologies just because of its decoding simplicity. This paper presents the performance evaluation of Alamouti’s space-time block coded (ASTBC) MIMO-OFDM systems covering channel model, channel capacity, coding scheme and diversity gain. The mathematical model of capacity is derived for deterministic, random and correlated Rayleigh fading channels. The channel capacity per unit bandwidth is evaluated as a function of SNR. It is observed that the channel capacity increases with the number of antenna added to the system due to the more diversity gain of Alamouti’s code. Finally we investigate the correlated MIMO fading channel with different correlation matrix. At the higher SNR value, independent and identically distributed (i.i.d) channel capacity outperforms the correlated channel capacity. But at very low SNR value correlated channel capacity outperforms the i.i.d channel capacity.

Keywords: MIMO, OFDM, ASTBC, Channel Capacity.

I. INTRODUCTION
The key challenge of future wireless communication systems is to provide high data rate wireless access at high quality of service. Since spectrum is a scarce resource and propagation conditions are hostile due to fading caused by destructive addition of multi-path components and interference from other users, it is required to radically increase spectral efficiency and to improve link reliability as a solution. During the last decade, many researchers have proposed multiple-input multiple-output (MIMO) wireless technology that seems to meet these demands by offering increased spectral efficiency through spatial multiplexing gain and improved link reliability due to antenna diversity gain [1, 2]. In addition, the MIMO system containing multiple antennas both at transmitter and receiver end can potentially meet the growing demand for higher capacity in wireless communications [3, 4]. The information capacity of wireless communication systems increases dramatically by using multiple transmitting and receiving antennas. Space-time coding, an effective approach for increasing data rate over wireless channels, employs coding techniques appropriate to multiple transmitting and receiving antennas. Hence, a new generalized complex orthogonal space time block code for several transmit antennas with full rate has been proposed in [5, 6].
In the fourth generation wireless communication systems the data rate may be as high as 1Gbps. For that, space-time coding techniques may be employed in conjunction with the multi-carrier code division multiple access (MC-CDMA) system to achieve very high data rate [7]. Different types of space-time trellis and block codes have been proposed for MC-CDMA systems in [8]. Many literatures has proposed space-time block coding schemes for orthogonal frequency division multiplexing (OFDM) systems based on the Alamouti’s scheme [9]. In frequency selective fading channels, space-time coded OFDM is a popular approach to provide transmit diversity and coding gains, which are termed as space-time trellis coded OFDM [10], and space-time block coded OFDM [11-16]. In particular, the combination of orthogonal space time block codes (OSTBCs) and OFDM, or simply OSTBC-OFDM has drawn much attention because it attains the maximum transmit diversity and has a simple maximum-likelihood (ML) receiver structure [12-16]. OFDM in conjunction with MIMO techniques allows us to realize and satisfy the ever growing demands of multimedia services and applications. OFDM has already been used successfully in standards for digital audio broadcasting (DAB), terrestrial video broadcasting (DVB-T), and wireless local area networks (WLANs) [17].

In this paper, we presented an Alamouti’s STBC coded MIMO-OFDM system for various antenna configurations to fulfill the demand of 4G wireless technology in section II. A brief discussion on Alamouti’s STBC is presented. Simulation results of Alamouti’s STBC with various antenna configurations are analyzed to evaluate the performance in terms of BER. In Section III, mathematical models of MIMO channel capacity for deterministic, random fading ergodic and correlated MIMO channels are presented and capacity of the different MIMO channels is evaluated for various antenna configurations under different channel correlation matrix. Simulation results are presented in Section IV and Conclusion in Section V.

II. SYSTEM MODEL

A. ASTBC-OFDM System Model

Consider a space time block coded MIMO-OFDM system [18] equipped with $N_T$ transmit antennas and $N_R$ receive antennas as illustrated in Figure 1. The message bit sequence is mapped into a sequence of BPSK symbols which will be converted into $N$ parallel symbol streams after serial-to-parallel (STP) conversion. Each of the $N$ parallel symbol streams is then encoded by the space-time block code (STBC) encoder into $\{X_i^{(t)}\}_{i=1}^{N_T}$, where $i$ is the antenna index and $t$ is the symbol time index. The number of symbols in a space-time codeword is $N_T = N_T \times T$. Then the symbol streams are subjected to inverse fast Fourier transform (IFFT) operation followed by cyclic prefix insertion between two consecutive OFDM symbols in order to reduce the effect of the delay spread of the multipath channels.

The length of the CP is adjustable and must be set in order to keep a bandwidth efficient system without occurring inter symbol interference or inter carrier interference. At the receiver, after removing the CP, and applying FFT, the transmitted symbol stream $\{X_i^{(t)}\}_{i=1}^{N_T}$ is estimated using the received signal $\{Y_j^{(t)}\}_{j=1}^{N_R}$. Assume the channel gain matrix $h_{ij}$ follows the Rayleigh distribution from the $i^{th}$ transmit antenna to the $j^{th}$ receive antenna over the $t^{th}$ symbol period. If the channel gains do not change during $T$ symbol periods, the symbol time index can be omitted and as long as the transmit antennas and receive antennas are spaced sufficiently apart, $N_T \times N_R$ fading gains $\{h_{ij}\}$ can be assumed to be statistically independent. If $X_i^T$ is the transmitted signal from the $i^{th}$ transmit antenna during $t^{th}$ symbol period, the received signal at the $j^{th}$ receive antenna during $t^{th}$ symbol period is given by equation (1), where $z_j^t$ is the noise.
process at the \( j^{th} \) receive antenna during \( i^{th} \) symbol period, which is modeled as the zero mean circular symmetric complex Gaussian (ZMCSCG) noise of unit variance, and is the average energy of each transmitted signal. In general we can write in equation (2)

\[
X = \begin{bmatrix} x_1 \\ -x_2^* \\ x_2 \\ y_1^* \end{bmatrix}
\]

Alamouti encoded signal is transmitted from the two transmit antennas over two symbol periods. During the first symbol period at \( t = T \), two symbols \( x_1 \) and \( x_2 \) are simultaneously transmitted from the two transmit antennas. During the second symbol period \( t = 2T \), these symbols are transmitted again, where \( -x_2^* \) is transmitted from the first transmit antenna and \( x_1^* \) transmitted from the second transmit antenna. For Maximum Likelihood signal detection of Alamouti’s space-time coding scheme, we assume that two channels gains \( h_1(H) \) and \( h_2(H) \) remain constant over two consecutive symbol periods such that

\[
h_1(t) = h_1(t+T) = h_1 = |h_1|e^{j\theta_1}
\]

\[
h_2(t) = h_2(t+T) = h_2 = |h_2|e^{j\theta_2}
\]

Where \( |h_1| \) and \( e^{j\theta_1} \) denote the amplitude gain and phase rotation over the two symbol periods. At the receiver the received signals \( y_1 \) and \( y_2 \) at time \( t \) and \( t + T_s \) can be given as

\[
y_1 = h_1 x_1 + h_2 x_2 + z_1
\]

\[
y_2 = h_1 x_2^* + h_2 x_1^* + z_2
\]

Where, \( z_1 \) and \( z_2 \) are additive noise at time \( t \) and \( t + T_s \) respectively. In this paper we have proposed Alamouti’s space time block code for two transmit antenna and more than one receive antenna case.
III. DETERMINISTIC CHANNEL CAPACITY OF MIMO-OFDM SYSTEM

For a MIMO system with $N_T$ transmit and $N_R$ receive antennas as shown in Figure 1, a narrowband time-invariant wireless channel can be represented by $N_R \times N_T$ deterministic matrix $H \in \mathbb{C}^{N_R \times N_T}$. Consider a transmitted symbol vector $X \in \mathbb{C}^{N_T \times 1}$ which is composed of $N_T$ independent input symbols, $x_1, x_2, x_3, \ldots, x_{N_T}$. Then, the received signal $y \in \mathbb{C}^{N_R \times 1}$, can be written in a matrix form as follows:

$$y = \sqrt{\frac{E_x}{N_T}} HX + Z$$

(5)

Where $Z = \{z_1, z_2, \ldots, z_{N_R}\}^T \in \mathbb{C}^{N_R \times 1}$, is a noise vector which is assumed to be zero mean circular symmetric complex Gaussian (ZMCSG). The autocorrelation of transmitted signal vector is defined as

$$R_{xx} = E \{XX^H\}$$

(6)

The capacity of a deterministic channel is defined as $C = \max_{f(x)} I(X; y)$ bits/channel use in which $f(x)$ is the probability density function (PDF) of the transmit signal vector $X$ and $I(x; y)$ is the mutual information of random vectors $x$ and $y$. From the fundamental principle of the information theory, the mutual information of the two continuous random vectors $X$ and $Y$ is given as

$$I(x; y) = H(y) - H(y|x)$$

(7)

In which $H(y)$ is the differential entropy of $y$ and $H(y|x)$ is the conditional differential entropy of $y$ when is $x$ given. Using the statistical independence of the two random vectors $Z$ and $X$ in Equation (5), we can write equation (7) as follows

$$I(x; y) = H(y) - H(z)$$

(8)

From the equation (8) we observe that $H(z)$ is a constant, we can see that the mutual information is maximized when $H(y)$ is maximized. Now, the auto-correlation matrix of $y$ is given as

$$R_{yy} = E \{yy^H\}$$

(9)

Putting the value of equation (5) in equation (9) we find

$$R_{yy} = \frac{E_x}{N_T} HXH^H + N_0 I_{N_R}$$

(10)

Where, $E_x$ the energy of the transmitted signals and $N_0$ is the power spectral density of the additive noise $\{z_i\}_{i=1}^{N_R}$. The differential entropy $H(y)$ is maximized when $y$ is ZMCSG which consequently requires $x$ to be ZMCSG. The mutual information can be found from equation (8) as follows

$$I(x; y) = \log_2 \det \left( I_{N_R} + \frac{E_x}{N_T N_0} HXH^H \right) \text{bps} / \text{Hz}$$

(11)

Then, the channel capacity of deterministic MIMO channel in the case of CSI known to both receiver and transmitter side is expressed as

$$C = \max_{I(R_{xx})=N_T} \log_2 \det \left( I_{N_R} + \frac{E_x}{N_T N_0} HXH^H \right) \text{bps} / \text{Hz}$$

(12)

When is not known at the transmitter side, one can spread the energy equally among all the transmit antennas so that the auto-correlation function of the transmit signal vector $x$ is given as

$$R_{xx} = I_{N_T}$$

Finally the channel capacity is given as

$$C = \log_2 \det \left( I_{N_R} + \frac{E_x}{N_T N_0} HH^H \right) \text{bps} / \text{Hz}$$

(13)

$$C = \sum_{i=1}^{r} \log_2 \left( 1 + \frac{E_x}{N_T N_0} \lambda_i \right)$$

(14)

Where $r = \min\{N_T, N_R\}$, denotes the rank of $H$ and $\lambda_i$ denotes the $i^{th}$ eigen value.

A. Ergodic Channel Capacity of MIMO-OFDM System
In above section, we have assumed that MIMO channels are deterministic where the channel gain remains constant. But in general case, MIMO channels change randomly and hence $H$ is a random matrix which means that its channel capacity is also randomly time varying and follows an ergodic process in practice. Then, we consider the following statistical notion of the MIMO channel capacity:

$$
\mathbb{C} = E \left\{ \max_{\rho(\alpha_\alpha = N_T)} \log_2 \det \left( I_{N_R} + \frac{E_x}{N_TN_0} H R_H H^H \right) \right\} \text{bps Hz}
$$

which is frequently known as an ergodic channel capacity. The ergodic channel capacity for the open-loop system without using CSI at the transmitter side from Equation (10) is given as

$$
\mathbb{C}_{OL} = E \left\{ \sum_{i=0}^{r} \log_2 \left( 1 + \frac{E_x}{N_TN_0} \lambda_i \right) \right\}
$$

Similarly, the ergodic channel capacity for the closed loop (CL) system using CSI at the transmitter side is given as

$$
\mathbb{C}_{CL} = E \left\{ \sum_{i=0}^{r} \log_2 \left( 1 + \frac{E_x}{N_TN_0} \lambda_i^\text{opt} \right) \right\}
$$

Sometimes the ergodic channel capacity is expressed as a function of the outage channel capacity. The outage probability can be defined as

$$
P_{\text{out}}(R) = P_e(C(H) < R)
$$

**B. Capacity of MIMO Correlated Fading Channel**

In general, the MIMO channel gains are not independent and identically distributed (i.i.d.) and the capacity of the MIMO channel are closely related to the channel correlation. For this reason, we consider the capacity of the MIMO channel when the channel gains between transmit and received antennas are correlated. We model the correlated channel as follows:

$$
H = R^{1/2}_r H_w R^{1/2}_l
$$

Where, $H_w$ denotes the independent and identically distributed (i.i.d) Rayleigh fading channel gain matrix and $R_l$ is the correlation matrix taking correlations between the transmit antennas, $R_r$ is the correlation matrix taking correlations between the receive antennas. Then the correlated channel capacity can be represented as

$$
C = \log_2 \det \left( I_{N_R} + \frac{E_x}{N_TN_0} R^{1/2}_r H_w R^{1/2}_l H_H R_H R^{1/2}_r \right)
$$

From the equation (19), we consider four cases for simulation. The correlation matrix can be given as in [24, 25].

**Case 1:** Correlation exists between transmit and receive antennas, transmit antennas and receive antennas but the correlation matrix $R_l$ and $R_r$ are identical.

**Case 2:** Correlation exists between transmit and receive antennas, transmit antennas and receive antennas but the correlation matrix $R_l$ and $R_r$ are not identical.

**IV. SIMULATION RESULTS**

Simulation results are presented to evaluate the performance of ASTBC coded MIMO-OFDM system with more than one antenna at the receiver and channel capacity for MIMO-OFDM system under independent, identically distributed (i.i.d) and spatially correlated MIMO-OFDM Rayleigh fading channels. This section is divided into two parts, i.e. the performance analysis of ASTBC coded MIMO-OFDM system and capacity analysis of deterministic, ergodic and spatially correlated MIMO-OFDM system under Rayleigh fading channel with different correlation matrix.
A. Performance Analysis of ASTBC MIMO-OFDM System

At first the performance of Alamouti’s Space Time Block Coded MIMO-OFDM system under Rayleigh fading channel is investigated with various antennas configurations. The simulation model employs BPSK modulation scheme and Alamouti’s coding scheme using two transmit antennas and more than one receive antennas.

Figure 2. BER Performance of Alamouti’s STBC for Various Antenna Configuration

In this case, we assume that transmit and receive antennas are uncorrelated, that is, those antennas are separated far enough from each other so that the fading processes affecting those antennas can be considered to be independent and identically distributed. We also assume that the channel coefficients are constant during each OFDM frame. Figure 2 compares the BER for different number of receive antennas and two transmit antennas system under Rayleigh fading channel. We observe that the performance of two transmit antennas with four receive antennas is much better than that of the system with two transmit antenna and less than four receive antennas in term of BER. We observe that the receive diversity gain increases with the number of receive antenna in the system and consequently the BER reduces quickly at the low value of SNR that improves the system performance of ASTBC MIMO-OFDM in terms of diversity gain.

B. Performance Analysis of MIMO Channel System

The performance of deterministic, ergodic and correlated MIMO channel capacity per unit bandwidth is investigated as a function of SNR. We also investigated the cumulative density function in the case of ergodic channel capacity. Finally the channel capacity of correlated MIMO channel for case 1 and case 2 as described above in section III (B) is investigated. Figure 3 and Figure 4 shows both deterministic channel capacity and MIMO ergodic channel capacity system increases with increasing the number of transmit and receive antennas in the system under consideration.

Figure 5 shows the capacity of i.i.d and correlated channel in terms of SNR with correlation exists between the transmit antennas and receive antennas but same correlation matrix. At 15 dB of SNR value 4×4 i.i.d channel provide 16.22 bps/Hz whereas 3×3 i.i.d channel provides 11.8 bps/Hz and 4×4 correlated channel provides 12.34 bps/Hz. Figure 6 shows the capacity of i.i.d and correlated channel in terms of SNR with correlation exists between the transmit antennas and receive antennas but different correlation matrix. In this case we notice that 4×4 i.i.d channel provide 22 bps/Hz whereas 4×4 correlated channel provides 14 bps/Hz. So i.i.d channel outperforms the correlated channel.
V. CONCLUSION

In this paper we evaluated the performance of the ASTBC MIMO-OFDM system under Rayleigh fading channel. We observe that the performance of two transmit antennas with more receive antennas is much better than that of the system with two transmit antenna and less receive antennas in term of BER due to the more diversity gain of Alamouti’s code. We also derived the mathematical model of deterministic, ergodic and correlated Rayleigh fading channel capacity where the CSI is perfectly known to the receiver and unknown to the transmitter side. We investigated the MIMO channel capacity with the various number of receive and transmit antennas. We observe that the channel capacity increases with the number of antenna added to the system. Finally we investigated the correlated MIMO fading channel with different correlation matrix. At the higher SNR value, i.i.d channel capacity outperforms the correlated channel capacity. But At very low SNR value correlated channel capacity outperforms the i.i.d channel capacity. It can be seen that MIMO-OFDM system can significantly increase the channel capacity of the system with the inclusion of more antenna to the system.

REFERENCES


