SOME APPROACHES AND PROPOSED TWO THEOREMS BASED ON INTUITIONISTIC FUZZY SET THEORY

Gyanendra Kumar Panda[1], Surendra Prasad Jena[2]
[1]Department of Mathematics B.I.E&T, Barapada, Bhadrak
[2]Department of Mathematics, S.B women's College , Cuttack, India
Email:gkpbdk@gmail.com[1], sp.jena08@gmail.com[2]

Abstract
In this paper, we present some approaches in IFSs which across some definitions, operations, properties and propose two theorems in IFSs.
Key words: algebra , fuzzy sets ,intuitionistic fuzzy sets.

1. Introduction

In 1965 zadeh's classical concept of fuzzy sets is a strongly mathematical tool to deal with the vagueness, introduced by [1]. Since the initiations of fuzzy set theory, there are suggestions for non-classical and higher order fuzzy sets for different specialized purposes. Out of several higher order fuzzy sets, intuitionistic fuzzy sets (IFSs) introduced by Atanassov is quite useful and applicable. IFSs are not fuzzy sets, although these are defined with the help of membership functions. But fuzzy sets are intuitionistic fuzzy sets. Every fuzzy sets has the form $\{\langle x, \mu_A(x), 1-\mu_A(x) : x \in X\rangle\}$ was introduced by[2,3]. In this section we present some approaches on IFSs viz: some definitions, basic operations, some properties and we extend two proposed theorems based on IFSs which was earlier introduced by [4].

In fuzzy set theory the in deterministic part is zero by assumption that full part of the degree of membership is determinism. But in real life situation it is not always so, and for such environment there is a need for the IFSs theory without any confusion. If deterministic part is zero, the IFSs theory coincides with fuzzy set theory.

2. Definitions of intuitionnstic fuzzy sets:-

Definition (2.1) Let us consider X be a non empty set and a fuzzy set A drawn from X is defined as A=$\{< x, \mu_{(A)}(x) >: x \in X\}$, where $\mu_A(x) : X \to (0,1)$ is the membership function on the fuzzy set A.

Definition (2.2) Let X be a nonempty set. An intuitionstic fuzzy set A in X is an object having the form

A= $\{< x, \mu_A(x), \vartheta_A(x) >: x \in X\}$, where the functions $\mu_A(x), \vartheta_A(x) : X \to [0,1]$. The degree of membership and the degree of non-membership of the element x$\in X$ to the set A, which is subset of 'X' and for every element x∈X, 0≤ $\mu_A(x) + \vartheta_A(x) \leq 1$. Further, we have $\pi_A(x) = 1 - \mu_A(x) - \vartheta_A(x)$ Called the intuitionistic fuzzy set index or hesita on margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of x$\in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e $\pi_A(x) : X \longrightarrow$ [0,1] and 0≤ $\pi_A \leq 1$ for every x∈ X.

Definition (2.3) (Equivalent IFS)

Two IFS P and Q are said to be equivalent to each other i.e p$\sim Q$ if there exists a function $f : \mu_P(x) \to \mu_Q(x)$ and $f : \vartheta_P(x) \to \vartheta_Q(x)$ both are injection and bijection, then the functions define a one- to- one correspondence between P and Q.

Definition (2.4) (Inclusive IFS)

Let P and Q are two IFSs, $P \subseteq Q \Rightarrow \mu_P \leq \mu_Q(x)$ and $\vartheta_p(x) \geq \vartheta_Q(x). \forall x \in X$, then P is a subset of Q and Q is the superset of P.

Definition (2.5) (Relations)

Let P,Q,R are IFSs, then

$\longrightarrow P \leq P$ i.e p is reflexive rel^n

$\rightarrow P \leq Q$ and Q$\leq P$ i.e symmetric rel^n

$\rightarrow P \leq Q$ and $Q \leq R \Rightarrow P \leq R$ i.e transitive rel^n.

$\Rightarrow$ If a relation is reflexive, symmetric and transitive, the rel^n is called "equivalence relation"

2.6 Some operations on Intuitionistic fuzzy sets

2.6.1 (Inclusion)

$A \subseteq B \leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\vartheta_A(x) \geq \vartheta_B(x)$, $\forall x \in X$.

2.6.2 (Complement)

$A^c = \{< x, \vartheta_A(x), \mu_A(x) >: x \in x\}$.

2.6.3 (Union)

A$\cup B = \{< x, max(\mu_A(x), \mu_B(x)), Min(\nu_A(x), \nu_B(x)) >: x \in X\}$.

2.6.4 (Intersection)

A$\cap B = \{< x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x)) >: x \in X\}$.

2.6.5 (Addition)

A$\oplus B = \{< x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x) >: x \in X\}$.

2.6.6 (Multiplication)

$A \otimes B = \{< x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \cdot \nu_B(x) >: x \in X\}$.

2.7 Properties of Intuitionistic fuzzy sets:

Suppose P, Q, R$\in$IFSs in X, then

i. $(P^c)^c = P$ i. e complimentary law.

ii. $p \cup p = p$ and $p \cap p = p$ i. e idempotent law.

iii. $(p \cup Q) \cup R = P \cup (Q \cup R)$ and $(P \cap Q) \cap R = P \cap (Q \cap R)$ i. e associative/ Qw

iv. $P\cup Q = Q \cup P$ and $P \cap Q = Q \cap P$ i. e commutative law.

v. $P\cup (Q \cap C) = (P \cup Q) \cap (P \cup R)$ And $P\cap (Q \cup C) = (P \cap Q) \cup (P \cap R)$ i. e distributive law.

Vi. $(P \cup Q)^c = P^c \cap Q^c$ and $(P \cap Q)^c = P^c \cup Q^c$(De. mogan's law.

Vii. $P\cap (P \cup Q) = P$ and $p \cup (P \cap Q) = P$ (absorption law)

viii. $P \oplus Q = Q \oplus P$ and $P \otimes Q = Q \otimes P$

ix. $P\oplus (Q \oplus R) = (P \oplus Q) \oplus R$ and $P\otimes (B \otimes C) = (P \otimes B) \otimes Ca$

x. $(P \oplus Q)^C = P^C \otimes Q^C$ $and (P \otimes Q)^C = P^C \oplus Q^C$

2.8 proposed theorems for Intuitionistic fuzzy sets:

Theorem 2.8.1: Let P,Q,R are three essential IFSs for which there are a, b, c $\in E$ such that $\mu_P(a) > 0, \vartheta(b) > 0$ and $U_R(c) > 0$, if $C(P) \subset I(Q)$G$\subset$ (R), $then$ the $real$ $numbers$ $\alpha, \beta, \gamma, \delta, \eta, \xi \in [0,1]$ So $that$ $J_{\alpha,\beta}(P) \subset H_{\gamma,\sigma}(Q) \subset Z_{\eta,\xi}(R)$.

Proof:

Let a set E be fixed then

For every IFS 'P', $C(P) = \{< x, K, L >: x \in E\}$

Where $K = \max_{a \in E} \mu_P(a), L = \min_{a \in E} \vartheta_P(a)$

And I(P)=$\{<x,k_1,l_1>:x\in E\}$

Where $k_1 = \min_{a \in E} \mu_P(a), l_1 = \max_{a \in E} \vartheta_P(a)$

Similarly $G(P) = \{< x, k_2, l_2 >: x \in E\}$

Where $k_2 = \min_{a \in E} \mu_P(a), \quad l_2 = \max_{a \in E} \vartheta_P(a)$

Let $C(P) \subset I(Q) \subset G(R)$, thus,

$$0 < \mu_P(a) \leq \max_{x \in E} \mu_P(x) = K \leq K_1 = \min_{x \in E} \mu_Q(x)$$

$$L = \min_{x \in E} \vartheta_P(x) \geq l_1 = \max_{x \in E} \vartheta(x) \geq \vartheta_Q(b) > 0.$$

And $0 < \mu_P(a) \leq \max_{x \in E} \mu_P(x) = K \leq k_2 = \min_{x \in E} \mu_Q(x)$

$$L = \min_{x \in E} \vartheta_P(x) \geq l_2 = \max_{x \in E} \vartheta_Q(x) \geq U_R(c) > 0$$

Let $P = \max_{x \in E} \pi_P(x) > 0, q = \max_{x \in E} \pi_Q(x) > 0$

And $r = \max_{x \in E} \pi_R(x) > 0$, since P, Q, R are essential IFSs.

Again, let $\alpha = \frac{k_1 - K}{2P}$ $\quad \beta = \frac{L + l_1}{2L}$

$\gamma = \frac{K + k_1}{2k_1}$ $\quad \sigma = \frac{L - l_1}{2q}$

$\eta = \frac{K + k_2}{2k_2}$ $\quad \xi = \frac{L - l_2}{2r}$

Then,

$$J_{\frac{(k_1-k)}{2P}, \frac{(L+l_1)}{2L}}(P) = \left\{< x, \mu_\rho(x) + \frac{k_1 - K}{2p}\pi_p(x), \frac{L + l_1}{2L}\vartheta_P(x) >: x \in E\right\}$$

$H_{\frac{(k_1+k)}{2k_1},\frac{(L-l_1)}{2q}}(Q)$

$$= \left\{< x, \frac{k_1+K}{2K_1}\mu_Q(x), \vartheta_Q(x) + \frac{L-l_1}{2q}\pi_Q(x) >: x \in E\right\}$$

$Z_{\frac{(k_2+K)}{2k_2},\frac{(L-l_2)}{2r}}(R)$

$$= \left\{< x, \frac{k_2+k}{2k_2}\mu_R(x), \vartheta_R(x) + \frac{L-l_2}{2r}\pi_R(x) >: x \in E\right\}$$

From

$$\mu_P(x) + \frac{k_1-k}{2P}\pi_P(x) \le K + \frac{k_1-k}{2P}P = \frac{k_1+k}{2} \le \frac{k_1+k}{2k_1}\mu_Q(x)$$

$$\vartheta_Q(x) + \frac{L-l_1}{2q}\pi_Q(x) \le l_1 + \frac{L-l_1}{2q}\cdot q = \frac{L+l_1}{2} \le \frac{L+l_1}{2L}\vartheta_P(x)$$

And $U_R(x) + \frac{L-l_2}{2r}\pi_R(x) \le l_2 + \frac{L-l_2}{2r}\cdot r = \frac{L+l_2}{2} \le \frac{L+l_2}{2K}\mu_P(x)$

Thus this establishing the theorem.

Theorem (2.8.2):

For every three IFSs P,Q and R, C(P) $\subset I(Q) \subset G(R)$, iff $there\ exists\ real\ numbers\ \alpha,\beta,\gamma \in [0,1] such\ that\ \alpha+\beta+\gamma \le 1\ and J_{\alpha,\beta,\gamma}(P) \subset H_{\alpha,\beta,\gamma}(Q) \subset Z_{\alpha\beta\gamma}(R)$.

Proof Let us consider C$(P\ ) \subset I(Q) \subset G(R)$

Thus K$\le k, L \ge l, and N \ge n.$

Where K= $\underset{x\in E}{max}\mu_P(x)$ L= $\underset{x\in E}{Min}\vartheta_P(x)$

K= $\underset{x\in E}{Min}\mu_Q(x)$ l= $\underset{x\in E}{Max}\vartheta_Q(x)$

And N= $\underset{x\in E}{Min}\vartheta_P(x)$ n= $\underset{x\in E}{Max}\vartheta_R(x)$

Let $\alpha = \frac{K+k}{2},\ \beta = \frac{L+l}{2}\ and\ \gamma = \frac{N+n}{2}$

From the above, we have

$$\mu_P(x) \le \frac{K+k}{2} \le \mu_Q(x)$$

$$\vartheta_P(x) \le \frac{L+l}{2} \ge \vartheta_Q(x)$$

$$\vartheta_Q(x) \le \frac{N+n}{2} \ge \vartheta_R(x)$$

For every x$\in$ E, it implies that

$$\text{Max}\left(\mu_P(x), \frac{K+k}{2}\right) = \frac{K+k}{2} = \text{Min}\left(\mu_Q(x), \frac{K+k}{2}\right)$$

$$\text{Min}\left(\vartheta_P(x), \frac{L+l}{2}\right) = \frac{L+l}{2} = \text{Max}\left(\vartheta_Q(x), \frac{L+l}{2}\right)$$

And $\text{Min}\left(\vartheta_P(x), \frac{N+n}{2}\right) = \frac{N+n}{2} = Max\left(\vartheta_R(x), \frac{N+n}{2}\right)$

$\Rightarrow J_{\alpha,\beta,\gamma}(P) \subset H_{\alpha,\beta,\gamma}(Q) \subset Z_{\alpha,\beta,\gamma}(R)$

Conversely, suppose $\alpha,\beta,\gamma \in [0,1] such\ that\ \alpha+\beta+\gamma \le 1\ and\ let\ J_{\alpha,\beta,\gamma}(P) \subset H_{\alpha,\beta,\gamma}(Q) \subset Z_{\alpha,\beta,\gamma}(R)$ then for every x$\in E$,

$$\left(max(\mu_P(x),\alpha) \le Min\left(\mu_Q(x),\alpha\right)\right)$$

$$\left(Min(\vartheta_P(x),\beta) \ge Max\left(\vartheta_Q(x),\beta\right)\right)$$

And $\left(Min(\vartheta_P(x),\gamma) \ge max(\vartheta_R(x),\gamma)\right), thus$

$$\underset{x\in E}{max} \max(\mu_P(x),\alpha) \le \underset{x\in E}{min} min\left(\mu_Q(x),\alpha\right)$$

$$\underset{x\in E}{min} min(\vartheta_P(x),\beta) \ge \underset{x\in E}{max} max\left(\vartheta_Q(x),\beta\right)$$

And similarly

$$\underset{x\in E}{min}\ min(\vartheta_P(x),\gamma) \ge \underset{x\in E}{max} max(\vartheta_R(x),\gamma)$$

Hence we have

$$\underset{x\in E}{max}\mu_P(x) \le max\left(\underset{x\in E}{max}\mu_P(x),\alpha\right) = \underset{x\in E}{max}\ max(\mu_P(x),\alpha) \le \underset{x\in E}{min}\ min\left(\mu_Q(x),\alpha\right)$$

$$= min\left(\underset{x\in E}{min}\mu_Q(x),\alpha\right) \le \underset{x\in E}{min}\mu_Q(x)$$

And $\underset{x\in E}{min}\vartheta_P(x) \ge min\left(\underset{x\in E}{min}\vartheta_P(x),\beta\right)$

$$= \underset{x\in E}{min}\left(min(\vartheta_P(x),\beta)\right) \ge \underset{x\in E}{max}\left(max\vartheta_Q(x),\beta\right)$$

$$= max\left(\underset{x\in E}{max}\vartheta_Q(x),\beta\right) \ge \underset{x\in E}{max}\vartheta_Q(x)$$

Similarly we have

$$\underset{x\in E}{min}\vartheta_P(x) \ge min\left(\underset{x\in E}{min}\vartheta_P(x),\gamma\right) = \underset{x\in E}{min}\ min(\vartheta_P(x),\gamma) \ge \underset{x\in E}{max}(max\vartheta_R(x),\gamma)$$

$$= max\left(\underset{x\in E}{max}\vartheta_R(x),\gamma\right) \ge \underset{x\in E}{max}\vartheta_R(x)$$

Therefore, $C(P) \subset I(Q) \subset G(R)$.
Which establishes the theorem.

Conclusion

In this paper we have presented some aspects on intuitionistic fuzzy sets such as definitions, operations, properties and study two useful theorems based on IFSs.

References

[1] L.A.Zadeh, fuzzy sets, inform and control 8 (1965) 338-353.

[2] K.Atanassov, intuitionistic fuzzy sets, fuzzy sets and systems 20(1) (1986) 87-96.

[3] K.Atanassov, intuitionistic fuzzy sets,VII ITKR'S session sofia, 1983.

[4] K.Atanassov, intuitionistic fuzzy sets, fuzzy sets and systems 110(2000) 267-269.

[5] K. De Supriya, R,Biswas, A.R.Roy, some operations , intuitionistic fuzzy sets, fuzzy sets and systems 114 (2000) 477-484.