



# MATHEMATICAL MODELING FOR TURBO LAG REDUCTION BY COMPENSATING THE AIR FLOW RATE OF TURBOCHARGED ENGINE

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## I. INTRODUCTION

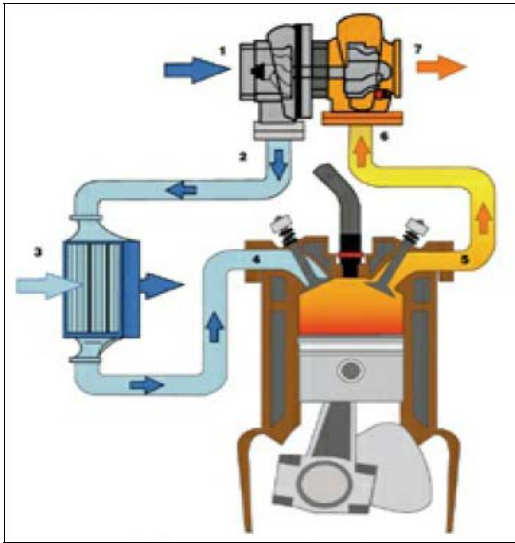
### Abstract

**In challenge for achieving the desired fuel economy benefits lies in optimizing the design and control of the engine boosting system. The demand for both reduced fuel consumption and increased performance has resulted in the current trend of downsizing and turbo-charging the combustion engine. One drawback of downsizing and turbo-charging is the effect of lag in boost pressure resulting in a decrease of vehicle performance. This is intensified at increased altitude. One way to reduce turbo lag is by operating the engine at a higher speed. In order to diminish lag, the launch device must allow for faster acceleration of the engine. By using the external power source, compressor is run for required period of time so engine gets boosted. In this work mathematical modeling is carried out for mass flow rate of turbine and compressor. This paper discusses a scalable modeling approach for the characterization of flow and efficiency maps for automotive turbochargers. The proposed approach is validated on a database of compressors and turbines for automotive boosting applications.**

**Keywords: Modeling, Response timing, boosting system**

A turbocharger is a turbine driven forced induction device that increases an engine's efficiency and power by forcing extra air into the combustion chamber. Simply turbocharger is a turbine, driven by exhaust gases that compress incoming air into the engine.

The hot side of the turbocharger receives its energy from the waste heat of exhaust system. The cold side of the turbocharger pressurizes fresh air from atmosphere and forces it into the engine. The pressure generated by the cold side is called the boost. The cold side is driven by a shaft that is connected to the hot side. This improvement over a naturally aspirated engine's output results because the turbine can force more air, and proportionately more fuel, into the combustion chamber than atmospheric pressure alone. After burning of fuel in combustion chamber, only the 1/3 of total power is useful for crankshaft. From total heat 1/3 heat loss to cooling water & 1/3 of total heat loss though exhaust. Hence turbocharger improves engine efficiency by using exhaust gas energy that would be otherwise lost. After burning of fuel in combustion chamber, only the 1/3 of total power is useful for crankshaft. From total heat 1/3 heat loss to cooling water & 1/3 of total heat loss though exhaust. Hence turbocharger improves engine efficiency by using exhaust gas energy that would be otherwise lost[1].



**Fig. 1 schematic diagram single cylinder of turbocharger: 1- Compressor, 2- compressor Discharge. 3-charge air cooler, 4-Intake valve, 5-Exhaust valve, 6-Turbine inlet, 7- Turbine discharge[2]**

## II. WORKING PRINCIPLE OF TURBOCHARGER

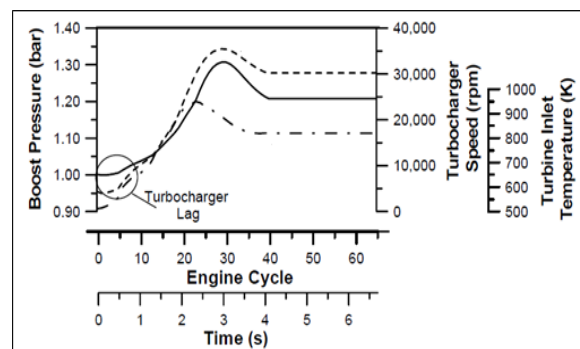
Turbocharger is the system which increases density of intake air. At the highest load engine produced high pressure and high temperature gases. So the turbine revolves at high speed and compressor gets high speed. After the air passes through the air filter it is transmitted to the turbocharger's compressor inlet. The air is then compressed, which increases its density by compressor section, and is discharged through a boost tube. In newly turbocharger cooling system is used, which had known as an intercooler or charge-air cooler. To receive the hot, boosted air as it leaves the compressor. The intercooler removes the excess amount of heat, which allows for further air density improvements. Another boost tube then transmitted the air from the intercooler discharge side to the engine intake manifold. Where the air is routed to the intake valves and it enters each cylinder [4]. Injection pressure and injection charge is to be changed, which is based upon density of intake air charge. So that proper combustion is occurred. After combustion, the exhaust passes through the exhaust valve and into the exhaust manifold. The exhaust manifold transmitted the exhaust gas energy in the form of pressure and heat into the turbine stage of the turbocharger. The turbine housing causes backpressure. This heat expands and backpressure against the turbine wheel blades causing the turbine shaft and

wheel to spin. So that it turn to drives the compressor wheel on the intake side of the system. The remaining exhaust then leaves the turbine stage and enters the exhaust system

## III. CONCEPT OF TURBO LAG

Turbocharger lag is the most prominent parameter of diesel engine transient operation. That drastically differentiates the torque pattern from the respective steady-steady conditions to study state condition. Turbocharger lag is produced because, although the fuel pump responds quickly to the increased fueling demand after a load or speed increase. The engine air-supply cannot match this higher fuel flow instantly but only after a number of engine cycles remaining to the inertia of the whole system. This phenomenon is enhanced by the disapproving turbocharger compressor characteristics at low-loads and low speeds. As a result of this slow reaction, the relative air-fuel ratio during the early cycles of a transient event assumes very low values which can be lower than stoichiometric[5].

During a typical load a detailed set of engine and turbocharger properties responses which increase transient event initiated from a step load change from 10 to 75% of full engine load. Engine having governor setting which is maintaining as constant. This is illustrated in Figure 2. It concerns a four-cylinder, four-stroke, medium-high speed, moderately turbocharged and after cooled, industrial diesel engine, rated at 241 kW at 1600 rpm. At the initial conditions, load torques and the engine are equal and the air-fuel ratio is relatively high due to the low loading. As soon as the new load is applied which is higher than initial one (this is accomplished in 1.3s), there is a significant shortage in the net torque, since the engine torque cannot instantly match its increased load counterpart, and so that the engine speed drops



**Fig.2: Turbocharger properties during a load increase transient event**

To avoid this turbo lag external source is used which is shown in fig.3. So that by using semi - supercharger it can manage the required power for engine. . So that high speed motor is used as external source. By using this motor speed of compressor is to be increases for starting condition. After gating the sufficient power from turbocharger motor is to be disengaging from compressor.

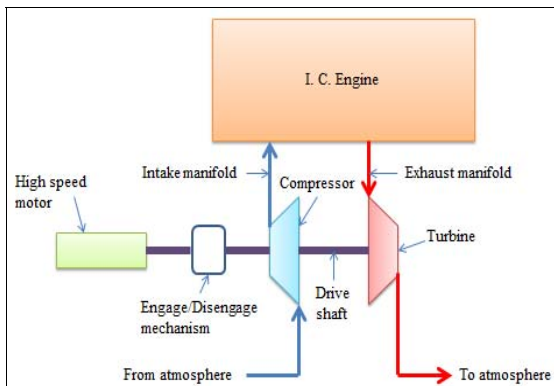


Fig.3: Schematic diagram of experimental setup

IV. MATHEMATICAL MODELING OF TURBOCHARGER

The use of behavioral models to extrapolate the flow and efficiency maps of automotive turbochargers is a common practice and several approaches have been proposed and developing virtual design studies, where the engine air path system is requires the ability to model the behavior of compressors and turbines while varying their key design parameters related to the geometric features of the stator and rotor [2].

According to the dimensional analysis theory for turbo machinery, it is possible to introduce dimensionless variables to reduce the number of overall variables representing the performance of compressors and turbines.

Table:1 Engine specification

Setup	Specification
Model	TATA Indica v2 LS
No. of cylinder	4
Volume	1405 cc
power	48.2 bhp@ 5000 RPM
torque	85 Nm @ 2500 RPM
Fuel Type	Diesel

Engine	Turbocharged
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The performance of a machine can be generally expressed as:

$$P_{out} = f(\dot{m}, p_{in}, T_{in}, \gamma, N, \nu, D, l_1, l_2, \dots)$$

$$\eta = f(\dot{m}, p_{in}, T_{in}, \gamma, N, \nu, D, l_1, l_2, \dots)$$

Where  $p$  is the pressure,  $\dot{m}$  the mass flow rate,  $T$  the temperature,  $N$  the rotational speed,  $\gamma$  the specific heat ratio,  $\nu$  the kinematic viscosity, and  $D, l_1, l_2$  represent design parameters that influence the performance.

A. Compressor mass flow

Firstly mass flow through the compressor from the atmosphere is to be calculated. Mass flow rate is indicated by  $\dot{m}_c$

$$\dot{m}_c = C_d A \frac{P_{in}}{\sqrt{T_{in}}} \left( \frac{P_{out}}{P_{in}} \right)^{\frac{\gamma}{\gamma-1}}$$

(1)

Where  $f\left(\frac{P_{out}}{P_{in}}\right) = \begin{cases} \frac{\gamma}{\gamma-1} \left[ \left( \frac{P_{out}}{P_{in}} \right)^{\frac{\gamma-1}{\gamma}} - \left( \frac{P_{out}}{P_{in}} \right)^{\frac{\gamma-1}{\gamma}} \right], & \text{if } \frac{P_{out}}{P_{in}} \geq \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \\ \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}, & \text{if } \frac{P_{out}}{P_{in}} < \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \end{cases}$

The discharge coefficient  $C_d$  is a function of the instantaneous valve lift to valve head diameter ratio,  $A$  is the geometric valve flow area and  $\gamma$  the specific heat capacities ratio; index  $p_{out}$  denotes downstream and  $p_{in}$  upstream Conditions[3].

B. Turbine mass flow

A database of flow and efficiency maps from different turbines of the same family was leveraged to define semi-empirical scalable models. The values of the diameter, trim and A/R ratio for the turbines considered in this study span, a range that should be summarized in to calibrate the parameters of the models [7].

The starting point to define the turbine flow model is the modified orifice equation, which is a generally accepted approach for modeling the turbine flow rate. This model characterizes the turbine flow rate as a non-isentropic expansion process through an orifice, based upon the equation:

$$\dot{m}_T = C_d A \frac{P_{in}}{\sqrt{T_{in}}} f(\lambda) \tag{2}$$

Where,

$$f(\lambda) = \begin{cases} \frac{\gamma}{\gamma-1} \left[ \left( \lambda^{\frac{\gamma}{\gamma-1}} - \left( \lambda^{\frac{\gamma}{\gamma-1}} \right) \right), & \text{if } \lambda \geq \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \\ \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}, & \text{if } \lambda < \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \end{cases}$$

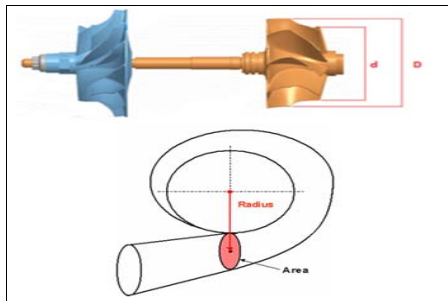
Where the polytropic coefficient  $m$  and the equivalent area  $C_d A$  are tunable parameters, and

$p_{in}$ ,  $T_{in}$  are reference values provided by the manufacturer.

The equivalent area  $C_d A$  in Equation generally relates to the throat area and the exit area of the turbine. Assuming that geometric similarity holds true, the throat area can be considered proportional to the turbine diameter and the A/R ratio  $\Gamma$ , while the exit area can be assumed proportional to the diameter and trim  $\tau$ . This leads to the following expression:

$$C_d A \cong a_1 \cdot (\Gamma D) + a_2 \cdot (D^2 \cdot \tau) + a_3 \quad (3)$$

Where the dimensionless coefficients  $a_1$ ,  $a_2$ ,  $a_3$  can be calibrated by using flow data from different design of turbines.



**Fig.4: Main design parameters for turbochargers compressors and turbines**

The starting point for this study is a set of turbocharger maps that was made available for a family of compressors and turbines. The available maps cover different combinations of three design parameters, figure 3 shows namely rotor diameter  $D$ , trim  $\tau$  and A/R ratio  $\Gamma$ . These parameters are defined in Equation

$$\text{Trim: } \tau = \left( \frac{D_c}{D_t} \right) \cdot 100$$

$$\text{A/R: } \Gamma = \frac{D_c}{D_t}$$

**Compressor A/R** - Compressor performance is comparatively very insensitive to changes in A/R. smaller A/R are used for high boost applications, and Larger A/R housings are sometimes used to optimize performance of low boost purpose. However, as this influence of A/R on compressor performance is minor, there are not A/R options available for compressor housings.

**Turbine A/R** - Turbine performance is greatly affected by changing the A/R of the housing, as it is used to adjust the flow capacity of the turbine. Using a smaller A/R will increase the exhaust gas velocity into the turbine wheel. This provides increased turbine power at lower engine speeds, resulting in a quicker boost rise. However, a small A/R also causes the flow to

enter the wheel more tangentially, which reduces the ultimate flow capacity of the turbine wheel

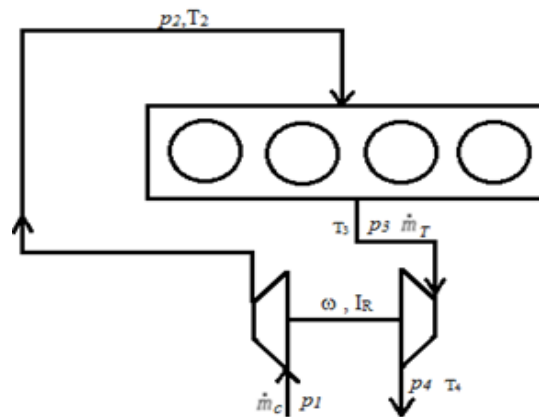
### V. TIME REQUIRED FOR BOOSTING TURBOCHARGER

The aim of a proper turbocharger matching leads to longitudinal dynamics as targeted of the propelled vehicle good efficiency. In more detail, the transient response of a turbocharger, thus turbine and compressor can be characterized while reviewing the following three main equations and their controllable state variables of a turbo-charged engine system as depicted in Figure 4. First, the formulation of the conservation of the angular momentum yields the gradient of angular velocity  $\omega = 2\pi \cdot n$  of the rotor [6] [10].

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{M_T - M_C}{I_R} \quad (4)$$

In equation (4)  $M_T$  represents the torque generated in the turbine,  $M_C$  the torque consumed by the compressor, and  $I_R$  denotes the rotational inertia of the rotor-system. Hereby  $I_R$  depends on the size and the corresponding diameter  $d$  of the rotor-system as

$$I_R \propto d^5$$



**Fig.5: Basic schematic of a turbo-charged engine**

From a mechanical point of view a high rotational acceleration,  $\omega$  can be achieved with a high level of work at the turbine, a low power consumption of the compressor, low level of friction and a low rotational inertia of the rotor system itself. This can be achieved by a small size of the rotor system or light-weight materials as is e.g. a *TiAl*-rotor

It is worthwhile mentioning that in reality the rotor cannot be considered as a rigid part. The treatment of the rotor as a flexible body helps to analyze the dynamic and corresponding

characteristics of the rotor in its floating ring bearings in more detail. Moreover, often the frictional loss in torque  $M_F$  in the turbocharger is not considered by the efficiency of the compressor,  $\eta_c$ . For the torque-producing at turbine side  $M_T$

$$M_T = \frac{\dot{m}_T}{\omega} \cdot \frac{k_2}{k_2 - 1} \cdot R_2 \cdot T_2 \cdot \eta_{T2} \cdot \left[ 1 - \left( \frac{p_2}{p_3} \right)^{\frac{k_2 - 1}{k_2}} \right] \quad (5)$$

$$X_1 = \dot{m}_T \cdot \frac{k_2}{k_2 - 1} \cdot R_2 \cdot T_2 \cdot \eta_{T2} \cdot \left[ 1 - \left( \frac{p_2}{p_3} \right)^{\frac{k_2 - 1}{k_2}} \right]$$

Therefore

$$M_T = \frac{X_1}{\omega} \quad (6)$$

For the torque-consuming compressor side ( $M_C$ ):

$$M_C = \frac{\dot{m}_c}{\omega} \cdot \frac{k_1 - 1}{k_1} \cdot R_1 \cdot T_1 \cdot \frac{1}{\eta_c} \cdot \left[ \left( \frac{p_1}{p_2} \right)^{\frac{k_1 - 1}{k_1}} - 1 \right] \quad (7)$$

Consider-

$$X_2 = \dot{m}_c \cdot \frac{k_1 - 1}{k_1} \cdot R_1 \cdot T_1 \cdot \frac{1}{\eta_c} \cdot \left[ \left( \frac{p_1}{p_2} \right)^{\frac{k_1 - 1}{k_1}} - 1 \right]$$

Therefore

$$M_C = \frac{X_2}{\omega} \quad (8)$$

In the light of a good efficiency, thus targeting a high  $M_T$  and a low  $M_C$ .  $M_T$  can be maximized while achieving a high gas mass flow of a high temperature, a low back-pressure level  $p_4$  and a high pressure level of  $p_3$ .  $k_1$  is polytropic exponent before compressor,  $k_2$  is polytropic exponent before turbine. Especially when looking at transient maneuvers, the former mentioned factors can, if they are shaped appropriately by the controlling strategy on a temporarily basis, ‘kick-off’ the rotational speed of the turbocharger without affecting the fuel economy significantly. By contrast,  $M_C$  should be a minimum while delivering a high level of  $p_2$ [11]. This can be achieved by a low level of pressure loss in the intake system before compressor accompanied by a low intake air temperature.  $T_4 \omega I_R \eta_c$

Now,

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{X_1 - X_2}{I_R \cdot \omega} \quad (9)$$

Integrating the both side of equation (9) with time limit  $t_1, t_2$  and angular velocity  $\omega_1, \omega_2$ .

$$\int_{t_1}^{t_2} d\omega = \int_{\omega_1}^{\omega_2} \left( \frac{I_R \cdot \omega}{X_1 - X_2} d\omega \right)$$

Here,  $I_R, X_1, X_2$  are constant functions.

$$\int_{t_1}^{t_2} d\omega = \frac{I_R}{(X_1 - X_2)} \int_{\omega_1}^{\omega_2} \omega \cdot d\omega$$

By integrating this question we get,

$$(\Delta t)_{\text{acceleration}} \cdot \omega = \frac{I_R}{(X_1 - X_2)} \left( \frac{\omega_2^2 - \omega_1^2}{2} \right) \quad (10)$$

Where  $\Delta t$  is time required to accelerating the vehicle from angular velocity of compressor  $\omega_1$  to  $\omega_2$ .

Non-adiabatic turbocharger operation can produce the turbo lag because the time required accelerating the turbocharger from angular velocity  $\omega_1$  to  $\omega_2$ . Turbocharger can't transfer whole power from turbine to compressor. Because errors are occurred like friction error, drive shaft error. Also at ideal condition exhaust gas not produced required amount of power to drive the compressor in efficient way [2].

### VI. TIME REQUIRED FOR BOOSTING TURBOCHARGER FOR REDUCED LAG

Turbocharger lag is reduced by using the external source or apply external power supply to compressor. High speed motor is used to supply external power. The compressor is run on motor for some time duration.  $M_s$  is torque applied by high speed motor to compressor.

Now,

$$\frac{d\omega}{dt} = \frac{X_1 - M_C + M_s}{I_R} \quad (11)$$

$$M_C = \frac{X_2}{\omega} \quad (12)$$

Put the value of equation (6), (8) in equation (11)

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{X_1 - X_2 + X_2 \omega}{I_R \cdot \omega} \quad (13)$$

Integrating the both side of equation (13) with time limit  $t_1, t_2$  and angular velocity  $\omega_1, \omega_2$ .

$$\int_{t_1}^{t_2} d\omega = \int_{\omega_1}^{\omega_2} \left( \frac{I_R \cdot \omega}{X_1 - X_2 + X_2 \omega} d\omega \right)$$

Here,  $I_R, X_1, X_2, X_2$  are constant functions.

$$\int_{t_1}^{t_2} d\omega = \frac{I_R}{(X_1 - X_2 + X_2 \omega)} \int_{\omega_1}^{\omega_2} \omega \cdot d\omega$$

By integrating equation we get,

$$(\Delta t)_{\text{acceleration}} \cdot \text{reduced lag} = \frac{I_R}{(X_1 - X_2 + X_2 \omega)} \left( \frac{\omega_2^2 - \omega_1^2}{2} \right) \quad (14)$$

Where, the  $\Delta t$  is the time required to boost the power.  $\omega_1, \omega_2$  are the angular velocities of turbine or compressor. By putting the value of  $X_1, X_2, X_2$

from equation and motor power we get the time required for boost the engine. Some losses are accrued in system during experimentation. These losses are not considering for modeling [5].

## VII. CONCLUSION

The work described in this paper focused on investigating the transient response of turbocharged engines. The study was conducted considering this problem from engine control perspective striving to identify how recent advances in control theory and implementation can help in reducing turbo lag from modern passenger cars. Since the beginning of the research work described here, the concept of engine downsizing has become a widely accepted method for reducing turbo lag from modern gasoline engines. Mathematical modeling can give the mass flow rate and time required for boosting of engine.

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## REFERENCES

- [1] R. Bontempo, M. Cardone, M. Manna , G. Vorraro, "Steady and unsteady experimental analysis of a turbocharger for automotive applications", *Energy Conversion and Management*, pp 72–80, April 2015
- [2] S. Shaaban and J. Seume, "Impact of Turbocharger Non-Adiabatic Operation on Engine Volumetric Efficiency and Turbo Lag", *Hindawi Publishing Corporation International Journal of Rotating Machinery*, Volume 2012, Article ID 625453, 11 pages, Dec 2011
- [3] Harald Stoffels, Stefan Quiring and Bert Pinggen, "Analysis of Transient Operation of Turbo Charged Engine", *SAE International Journal of Engine*, Volume 3, Issue 2, Sept 2010.
- [4] Weiguo Zhang, Mac Lynch, Robert Reynolds, "A Practical Simulation Procedure using CFD to Predict Flow Induced Sound of a Turbocharger Compressor", *SAE International Passeng. Cars - Mech. System* Volume 8, Issue 2 , pp521-525 July 2015
- [5] Ruixue Lil, Ying Huang, Gang Le, He Song, "Control-oriented Modeling and Analysis for Turbocharged Diesel Engine System", *2nd International Conference on Measurement, Information and Control*, pp 855-860, 2013
- [6] Constantine D. Rakopoulos, Evangelos G. Giakoumis, " Diesel Engine Transient operation", *Book published in Springer*, 2009
- [7] Philippe Moulin and Jonathan Chauvin, "Analysis and control of the air system of a turbocharged gasoline engine", *47<sup>th</sup> IEEE conference on decision and control*, pp 5641-5649, Dec 2008.
- [8] S. Shaaban1 and J. Seume, "Impact of Turbocharger Non Adiabatic Operation on Engine Volumetric Efficiency and Turbo Lag", *International Journal of Rotating Machinery*, Article ID 625453, 2012.
- [9] Marcello Canova, Fabio Chiara, Giorgio Rizzoni , "Design and Validation of a Control-Oriented Model of a Diesel Engine with Two-Stage Turbocharger", *SAE International Journal Engines*, Volume 2, Issue 2, pp 387-397 ,Jan 2009
- [10] S.Sunil Kumar Reddy, Dr. V. Pandurangadu, S.P.Akbar Hussain, "Effect of Turbo charging On Volumetric Efficiency in an Insulated Di Diesel Engine For Improved Performance", *International Journal of Modern Engineering Research (IJMER)*, Vol.3, Issue.2, pp-674-677, April 2013.
- [11] Philip Severyn, Jeff Hemphill, Philip George and Todd Sturgen, "Launch Devices for Turbo-Charged Engines", *SAE International J. Passenger. Cars - Mech. System*, Volume 5, Issue 1, Jan 2012.
- [12] Mehdi Nakhjiri, Peter Pelz and Berthold Matyschok , "Physical Modeling of Automotive Turbocharger Compressor: Analytical Approach and Validation", *SAE International Journal*, Sept 2011.