ANALYSIS OF PHOTONIC CRYSTALS
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Abstract
This paper discusses analysis of photonic crystal (PhC). A comparative study of PhC structure has been presented using triangular and square lattices. The crystal features a periodic arrangement of pillars positioned in air (or holes embedded in a solid dielectric material) at optimal spacing. Depending on the geometrical parameters, waves within a certain frequency range will be blocked from propagating through the crystal, creating a photonic bandgap. The bandgap thus created, has been analyzed by modifying the parameters of PhC like material of the crystal, radius of the pillars (holes), etc.

Index Terms: Photonic Crystal, photonic bandgap.

I. INTRODUCTION
An intense research on photonic crystals has been going on from the past few decades [1-23]. As there is a run towards use of THz frequency, it poses a need for designing all optical components. One such class of material which support building optical components is called as “photonic crystal”. The basic concept behind these photonic crystal materials stem from the pioneer paper in the field by Sajeev John and Yablonovitch. In a nutshell, photonic crystal is a material that can affect the properties of photons, in a similar manner as electrons are affected by atomic lattices of a semiconductor. Thus, we can say PhC acts as an optical insulator.

II. LITERATURE REVIEW
Lord Rayleigh in 1887 has discussed, the propagation of waves in an infinite laminated medium and already had the finite bandgap for 1D crystals. Almost after 100 years, in 1987 high refractive index contrast and multi dimensionality PhC were proposed [1]. Since then, lots of research work has been done in the field but as the dimensions of the design fall on the scale of micrometer and nanometer practically fabricating them was a bit tedious. Various 2D, quasi 2D [2], 3D structures were practically designed and fabricated [3-4]. PhC were then designed as waveguides [5-6] micro-cavities, multi-mode optical fiber[7], electro-optic switches [8], sensors [9], optical gates [10], etc. Also work on crossing the optical waveguides with minimal crosstalk was done [11]. Currently research on PhC based frequency selective polarizer’s, frequency selective filters, etc. is going on[12-13]. Also a study on transmission of slow light through PhC is been done by several scientists and research scholars [14-15]. The rapid progress in the field shows possibility of building a completely optical integrated circuit similar to semiconductor IC’s available in market today.

The remainder of the paper is organized as below. Section III comprises of a brief theory on photonic crystals, their designing and photonic bandgap structure followed by various algorithms used to compute bandgap in section IV. The effect of different physical parameters on PBG with experimental results are discussed in the section V.

III. THEORY BEHIND PHOTONIC CRYSTAL
Photonic crystals are periodic structures constructed using alternating layers of high and low dielectric structures. Depending on periodicity in its dimensions, PhC are broadly
classified as 1D, 2D and 3D crystals as shown in Fig.1. Photons behaving as wave do (or do not) propagate through the PhC depending upon the relation between its physical dimension and the wavelength of the propagating wave. Allowed wavelengths that propagate through the crystal form a band called photonic band. Group of wavelengths that are blocked from propagating through the crystal form a photonic bandgap (PBG).

To understand the light propagation in the crystal we need to understand behavior of light in periodic dielectric media. Maxwell’s equations are as-

\[ \nabla \times E = -\frac{1}{\epsilon} \frac{\partial H}{\partial t} \]  

\[ \nabla \times H = \frac{4\pi}{c} J + \frac{1}{\epsilon} \frac{\partial E}{\partial t} \]  

\[ \nabla \cdot E = 4\pi \rho \]  

\[ \nabla \cdot H = 0 \]

Where, \( E \) & \( H \) are electric and magnetic field respectively, \( \epsilon \) is dielectric function, \( J \) is free current density and \( \rho \) is free charge density. Assuming \( \epsilon \) is independent of frequency, it does not absorb light and hence can be treated as real number. As we are not interested in any free charges or current \( J = \rho = 0 \).

Then equation (2) becomes

\[ \nabla \times H = \frac{1}{\epsilon} \frac{\partial E}{\partial t} \]

Substituting for \( E \) from equation (5) in equation (1), we obtain,

\[ \nabla \times \frac{1}{\epsilon} \nabla \times H = \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} \]

Where, \( \nabla \times \frac{1}{\epsilon} \nabla \) is the hermitian operator. Converting equation (6) to time-harmonic state (i.e. time dependency \( \frac{\partial^2}{\partial t^2} \) to some angular frequency),

\[ \nabla \times \frac{1}{\epsilon} \nabla \times H = \left( \frac{\omega}{c} \right)^2 H \]

Eigen-operator is hermitian implies that eigenfrequencies \( \omega \) in equation (7) are real. Applying Bloch-Floquet theorem to the above equation (7) solves to

\[ H = e^{i(kx-\omega t)} \cdot H_k \]

Where, \( H_k \) is a periodic function of position and hence eigenproblem can be considered over a finite domain. This implies that eigenfrequencies \( \omega \) are a countable function of the wave vector \( k \); these frequency bands form the band structure of the crystal. Because \( k \) is only unique to multiples of \( \frac{2\pi}{a} \) then it is sufficient to solve the eigenproblem for a finite range of \( k \) \([-\frac{\pi}{a} \ldots \frac{\pi}{a}] \) called First Brillouin Zone. If \( k \) and \( -k \) produces same result then the range can be reduced to \([0 \ldots \frac{\pi}{a}] \) called irreducible Brillouin zone and in three dimension is commonly given by vertices \( \beta, X, M, K, \Gamma, \) etc. For more detailed discussion refer [11],[16]. Since we are interested in only the extrema of the frequency bands, it is usually sufficient to consider only those wavevectors which are along the edges of the irreducible Brillouin zone. This allows us to plot a 2D graph of photonic band structure.

IV. ALGORITHM TO COMPUTE PHOTONIC BANDGAP

To design a photonic crystal, it is essential to engineer the location and size of the bandgap [16]. Various numerical modeling methods used to compute photonic bandgap are Plane Wave Expansion (PWE) method, Finite Element Method (FEM), Finite difference Time Domain Method (FDTD), Transfer Matrix Method (TMM), etc [17]. To perform rigorous numerical analysis using any of these algorithms there are several free and commercial platform available [18].

FEM technique requires the entire volume of the geometry to be meshed. However, each mesh element can differ in material properties from their neighboring elements. FEM excels at modeling complex inhomogeneous configurations. FDTD technique also requires entire volume to be meshed. This mesh must be uniform, so that the mesh density is determined by the smallest detail of the configuration.
Unlike FEM, FDTD works in time domain making them suitable for transit analysis problems [19].

TMM yields the amplitude of the electromagnetic field of monochromatic waves reflected by and transmitted through the structure. The solution is achieved through propagation of the fields in the homogenous layers, and the continuity of the tangential components of the electric and magnetic field at the interfaces. If the field is known in the beginning of a layer, the field at the end of the layer can be computed from a simple matrix operation [17].

PWE method was used to obtain rigorous solution of Maxwell’s equations. Assuming discrete transitional symmetry and therefore infinite extension of the modulation, the field decomposes into periodic modes. A solution is obtained by expansion in zones of the periodic distribution of refractive index with periodic functions and by energy conservation at boundaries.

V. EFFECT OF PARAMETERS ON PBG
A. Effect of radius of the pillar in a square lattice.

Consider wave propagation in a photonic crystal consisting of GaAs rods standing in air. The band structure is computed using FEM method. As it is a repetitive pattern only one pillar is simulated using Floquet-periodic boundary condition. Radius of the GaAs pillar is varied from 55nm to upto 85nm. The length of the side of primitive cell is taken to be 375nm. Fig. 2 shows the geometry of the primitive cell.

![Fig. 2 Geometry of the primitive cell with side length 375 nm, GaAs pillar radius = 70 nm surrounded by air.](image)

Table 1 shows the plot of electric field of z-component around first band and the photonic bandgap, for different radius. It is seen that as the radius increases, maxima and minima of z-component of the electric field increases. Further increase in the value of radius will increase the number of maxima and minima occurring. Also, with increase in radius, bands shifts towards lower frequencies, width of the bandgap reduces, while number of bands increases.

B. Effect of radius of the pillar in a triangular lattice

Consider a 2D PhC of cylinders standing in air arranged in triangular lattice [16]. The effect of physical parameters of PhC on the photonic bandgap are discussed in references [20-22]. The normalized radius of the cylinders is varied from 0.25 to 0.5 and photonic bandgap is plotted for both E and H polarization using plane wave expansion (PWE) method.

It is observed that as radius of the cylinder increases higher order bands in the E-polarization increases but band strength decays in H- polarizations. Inversely, when radius of the cylinder is reduced PBG in H- polarization enlarges while those in E – polarization decays. Hence to obtain a complete bandgap in both E and H polarization there has to be trade off maintained and appropriate radius of the cylinders has to be chosen.

![Fig. 3 photonic bandgap for radius= 0.5(a)E-pol (b)H-pol.](image)
C. Effect of change in material properties

The contrast in the refractive index of the substrate (slab) and the cylinder have considerable effect on the PBG. It is observed that the PhC has gap of larger width when there is high contrast in R.I. of the material.

VI. CONCLUSION

To obtain a complete three dimensional bandgap irrespective of polarization, high and low dielectric material should be completely isolated islands yet should be connected by thin veins, i.e, thin veins of high dielectric material surrounding lower dielectric material (eg. larger air holes and thinner veins of silicon). Also, the larger the contrast between high and low dielectrics will fetch a wider bandgap. Crystals having almost spherical Brillouin zone have greater chance to have complete bandgap. The gap arises only for wavelength that correspond to crystal periodicity.

VII. ACKNOWLEDGMENT

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Table 1: Variation in bandgap and eigenproblem due to change in radius.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Z component of the Electric field (V/m)</th>
<th>Photonic bandgap structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>85nm</td>
<td>Eigenfrequency=3.863E14 (1) Surface: Electric field, z component (V/m)</td>
<td><img src="image1.png" alt="Graph" /></td>
</tr>
<tr>
<td>70nm</td>
<td>Eigenfrequency=4.2845E14 Surface: Electric field, z component (V/m)</td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>55nm</td>
<td>Eigenfrequency=4.3472E14 Surface: Electric field, z component (V/m)</td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

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