Abstract
In this research paper, Image segmentation technique based on maximum fuzzy entropy function and optimization is applied on Magnetic Resonance (MR) type of brain images to detect various brain malignancy is presented. The proposed method accomplishes image segmentation centered on optimal thresholding of the input malignant MR brain images. The input MRI of malignant brain is classified into two Membership Functions (MF), whose MFs of the fuzzy region is Z-function and S-function. The optimization of fuzzy MFs parameters to identify the proper threshold is obtained by means of an algorithm called Modified Particle Swarm Optimization (MPSO). The maximum fuzzy entropy is considered to be the objective for finding the optimal value among the fuzzy MFs parameter. From the course of various examples, the performance of the proposed MPSO is compared with those using existing entropy-based object segmentation methods and the supremacy of the proposed Modified PSO method is validated. The simulated experimental results are compared with the exhaustive/enumerative search method and standard Otsu segmentation technique. The result indicates the proposed integrated fuzzy entropy-optimization approach in segmenting malignancy from brain achieves proper segmentation with minimum time for computational analysis.

Keywords: Maximum-Fuzzy entropy, Membership Function, Modified Particle Swarm Optimization

1. Introduction
The most significant and highly complicated low-level image analysis and processing errands are image segmentation. The image segmentation is a tedious process to extract region of interest or specified meaningful objects from an image based on specific threshold levels. Various research studies in image processing describes threshold based segmentation is most effective [1-4]. The idea to separate a desired object/area from the background of an image in applying different threshold values always remains as a dare [5-6]. From the histogram of any input image the maximum entropy function can be derived [7][8]. Among the most well-known thresholding methods, fuzzy entropy-based method is vastly studied and considered to be effective [8]. The fuzzy entropic correlation function defined by Yen et al [9] tries to obtain an optimum threshold that maximizes the entropy. The research works described in [10] [11] inferred the maximization of the Renyi’s entropy and minimisation cross entropy, computed from auto correlation functions are to set a threshold value to characterize the segmentation of region of...
interest from an image. The most significant role played by fuzzy sets in deploying systems with their capability to model non-statistical imprecision is discussed in [12][13]. Luca and Termin [14] introduced the concept of fuzzy entropy as function on fuzzy sets that converges to very low value when the sharpness of its fuzzy set argument is enhanced. There have been numerous applications of fuzzy entropies in image segmentation. The thresholding method based on the fuzzy relation along with maximum fuzzy entropy principle using different fuzzy partitions on a two-dimensional histogram has been conferred by Cheng et al [15]. An optimum threshold is set among the least sum of entropies for test image and the significance of highlighting fuzzy MF’s in indicating depth of gray value from the background of an image is discussed in [16].

The function of probability partition and fuzzy c-partition was discussed in [17] to measure the compatibility among these two. A novel optimal methodology to segment malignancy from input malignant brain image based on the maximum fuzzy entropy via probability analysis, using fuzzy partition function and maximum entropy logic is defined [23]. The image is partitioned on bi-levels, hence two classes or areas, namely the dark class and the gray class, where Z membership function corresponds to dark category and S membership function corresponds to bright category of pixels in the input test image [24].

This research examines the performance of segmentation techniques applied on malignant MR brain images using standard Otsu method [18] and enumerative/exhaustive fuzzy entropy object segmentation method. The optimal values on fuzzy membership functions can only give the apt threshold value. For that, it is required to search all the possible fuzzy MF combinations. Thus, the segmentation problem is formulated as an objective of optimization problem. It is proved from various researches that particle swarm optimization (PSO) can deploy acceptable result for many engineering problems [19-21]. Hence particle swarm optimization (PSO) approach extends effective way to find optimal fuzzy membership parameters for segmentation.

This paper explores the strength of MPSO in finding the optimal fuzzy MFs parameters to find the threshold value and maximum fuzzy entropy in the malignant MR brain image. This paper is ordered as follows. In Section 2, for the integrity, a simple description is given in object segmentation method based on probability analysis function and fuzzy entropy function, which is similar to the method presented in [5] [21][22]. In Section 3, the modified PSO approach to find the optimal combination of all fuzzy MF parameters is presented. In Section 4, we evaluate the performance of the proposed entropy based thresholding approach using malignant MR brain images and compare it with leading techniques from the literature. And finally, Section 5 concludes this paper with its future application.

2. Background

2.1 Input Image as a Fuzzy Event

Consider an input image \( A \) of size \( M \times N \) with \( L \) gray levels ranging from \( L_{\text{min}} \) to \( L_{\text{max}} \). Let \( a_{ij} \) denote the gray level of the image \( A \) at the \((i,j)\)th pixel [23-25]. The histogram of the image is denoted as \( h_k \) and is defined as

\[
h_k = \frac{n_k}{M \times N}, k = 0, ..., L-1
\]

where, \( n_k \) denotes the number of occurrences of gray levels in input image \( A \). An image can be modelled by a triplet \((G,K,P)\) factors, where, \( G = \{g_0, g_1, g_2, ..., g_{L-1}\} \), \( P \) is the probability measure of the occurrence of gray levels, i.e. \( Pr\{g_k\} = h_k \).

A probability space based fuzzy event can be modeled for an image. In accordance with the fuzzy set theory, the input image \( A \) can be transformed into an array of fuzzy singletons \( S \) by a membership function [5][24][25].

\[
S = \{\mu_A(a_{ij}) | i = 1, 2, ..., M; j = 1, 2, ..., N \}
\]

Then, the degree of some properties of the input image such as brightness, darkness, etc. possessed by the pixels \((i,j)\) can be denoted by the membership function \( \mu_A(t_{ij}) \) of the fuzzy set, \( A \in G \) [24][25]. In fuzzy set notation, image \( A \) can be written with its belonging as

\[
A = \frac{\mu_A(t_{1j})}{r_1} + \frac{\mu_A(t_{2j})}{r_2} + \ldots + \frac{\mu_A(t_{kj})}{r_k}
\]
\[ A = \sum_{k} \mu_{A}(r_{k}) / r_{k} \]  
(4)

Here “+” indicates union of the sets in fuzzy logics.

The Equation to obtain the probability of input image \( A \) is given as

\[ \sum_{r_{k}} \mu_{A}(r_{k}) \text{Pr}(r_{k}) \]  
(5)

and the Equation corresponding to conditional probability function for the input image tends to be,

\[ p[r_{k} | A] = \mu_{A}(r_{k}) / \text{Pr}(A) \]  
(6)

Fuzzy entropy function describes the fuzziness of a fuzzy set for the given input image. The entropy function with fuzzy is a measure of the uncertainty of a fuzzy set. The domain of the input image be given as \( Z = \{(i, j): i = 0, 1, 2, \ldots, M - 1; j = 0, 1, 2, \ldots, N - 1\} \)
(7)

and the gray level of the input image as \( G = \{0, 1, \ldots, L - 1\} \) where \( M, N \) and \( L \) are three positive integers [25]. If the gray level value of the input image at the pixel \( (x, y) \)
is \( A(x, y) \) then

\[ Z_{k} = \{(x, y): A(x, y) = k, (x, y) \in G\}, k = 0, 1, \ldots, L - 1 \]  
(8)

Let the threshold of the input image \( A \) be \( T \) that segments an image into its target region and background. The domain \( Z \) of the original image can be classified into two different groups, \( F_{d} \) and \( F_{b} \), which comprises of pixels with low gray levels of the image and high gray levels of the image, respectively. An unknown probabilistic partition of \( Z \) denoted as \( \Pi_{2}\{F_{d}, F_{b}\} \)
describes its probability distribution function as

\[ p_{d} = p(F_{d}) \]  
(9)

\[ p_{b} = p(F_{b}) \]  
(10)

For an image with 256 gray levels, \( \mu_{b} \) and \( \mu_{d} \) indicates the membership functions that corresponds to the bright and dark pixels. Let \( a, b \) and \( c \) be the three parameters of the fuzzy membership function, which means that the threshold \( T \) depends on \( a, b \) and \( c \) values [22]. Consider

\[ Z_{kd} = \{(x, y): f(x, y) \leq T, (x, y) \in Z_{k}\} \]  
(11)

\[ Z_{kb} = \{(x, y): f(x, y) > T, (x, y) \in Z_{k}\} \]  
(12)

for each \( k = 0, 1, \ldots, 255 \).

Then the following Equations hold:

\[ p_{kd} = p(Z_{kd}) = p_{k} \ast p_{dk} \]  
(13)

\[ p_{kb} = p(Z_{kb}) = p_{k} \ast p_{bk} \]  
(14)

For input image, the conditional probability of a pixel, obviously set as \( p_{dk} \) and \( p_{bk} \) need to be categorised into the ‘dark’ class and ‘bright’ class, by means of the constraint, where the pixel belongs to \( D_{k} \) with

\[ p_{dk} + p_{bk} = 1, (k = 0, 1, \ldots, 255) \]  
(15)

The grade of pixels in the input image is classified into class ‘dark’ and class ‘bright’, where the gray level value \( k \) is found to be equal to its conditional probability function \( p_{dk}, p_{bk} \), respectively [3][15][16][18][24]. The equations for probability function \( p_{d} \) and \( p_{b} \) hold as follows:

\[ p_{d} = \sum_{k=0}^{255} p_{k} \ast p_{dk} = \sum_{k=0}^{255} p_{k} \ast \mu_{d}(k) \]  
(16)

\[ p_{b} = \sum_{k=0}^{255} p_{k} \ast p_{bk} = \sum_{k=0}^{255} p_{k} \ast \mu_{b}(k) \]  
(17)

2.3 Formation of Fuzzy Membership function for input image classification

The two Fuzzy MFs, \( S \)-function and \( Z \)-function are applied to classify the brain input image for calculating the fuzzy entropy function which is shown in Figure 1. In this work, \( Z(k, a, b, c) \) - function denotes the MF \( \mu_{d}(k) \) of the class of ‘dark’ pixels and \( S(k, a, b, c) \) - function denotes the MF \( \mu_{b}(k) \) of the class of ‘bright’ pixels. The fuzzy MF parameters \( a, b, c \) satisfy the constraint \( 0 \leq a \leq b \leq c \leq 255 \). The Equation (18) shows the MF of \( Z(k, a, b, c) \). The Equation (19) shows the MF of \( S(k, a, b, c) \).

\[ \mu_{d}(k) = \begin{cases} 
1, & k \leq a \\
1 - \frac{(k-a)^{2}}{(c-a)(b-a)}, & a < k \leq b \\
\frac{(k-c)^{2}}{(c-a)(c-b)}, & b < k \leq c \\
0, & k > c 
\end{cases} \]  
(18)

\[ \mu_{b}(k) = \begin{cases} 
1 - \frac{(k-a)^{2}}{(c-a)(b-a)}, & a < k \leq b \\
\frac{(k-a)^{2}}{(c-a)(c-b)}, & b < k \leq c \\
1, & k > c 
\end{cases} \]  
(19)

For the class of dark pixels, Fuzzy entropy function, \( H_{dark} \) is calculated based on Equation (20) and for the class of bright pixels the fuzzy entropy function, \( H_{bright} \) is
calculated based on Equation (21) as shown below:

\[
H_{\text{dark}} = -\sum_{k=0}^{255} \frac{p_k \cdot \mu_d(k)}{p_d} \log\left(\frac{p_k \cdot \mu_d(k)}{p_d}\right)
\]

(20)

\[
H_{\text{bright}} = -\sum_{k=0}^{255} \frac{p_k \cdot \mu_b(k)}{p_b} \log\left(\frac{p_k \cdot \mu_b(k)}{p_b}\right)
\]

(21)

Then the total fuzzy entropy function \(H(a,b,c)\) is given as

\[
H(a,b,c) = H_{\text{dark}} + H_{\text{bright}}
\]

(22)

This total fuzzy entropy function depends on the fuzzy parameters - \(a,b,c\). The blend of these three parameters is chosen such that the total fuzzy entropy function \(H(a,b,c)\) attains a maximum value [24]. The Equation to segment the image into classes of dark and bright, via appropriate threshold is as follows:

\[
\mu_{\text{d}}(T) = \mu_{\text{b}}(T) = 0.5
\]

(23)

Figure 1. MF graph showing the intersection of \(Z\) membership function for dark class- \(\mu_d(k)\) and \(S\) membership function for bright class- \(\mu_b(k)\) at threshold \(T\)

From the Figure 1, it is evident that threshold \(T\) is the point of intersection of \(\mu_d(k)\) and \(\mu_b(k)\). The solution to derive \(T\) can be obtained from the Equation (24) [24]. Hence the threshold \(T\) can be easily formed by:

\[
T = \begin{cases} 
\frac{a + \sqrt{(c-a)(b-a)}}{2}, & (a+c)/2 \leq b \leq c \\
\frac{c - \sqrt{(c-a)(c-b)}}{2}, & a \leq b \leq (a+c)/2 
\end{cases}
\]

(24)

2.4 Modified Particle Swarm Optimization (MPSO) Algorithm Approach on optimization

In the early 1990’s research on optimization on population based, stochastic search technique has been developed by Eberhart and Kennedy called PSO [19][21]. The search process of PSO algorithm was highly inspired by social nature and behaviours of animals which include flocking of birds, fish schooling etc [24]. PSO algorithm begins with the basic random initialization of a population in the specified search space for the given problem. This algorithm deals with the working of the social behaviour of particles in the swarm population [25]. The most remarkable are its characteristics of stable convergence, through which it can generate a high quality solution in a shorter execution or computation time rather than other stochastic methods. The main concept of modification of a search point by PSO is shown in Figure 2.

Figure 2. Concept of modification of a search point by PSO

where \(n x_i^d\) is the particle’s current position, \(n+1 x_i^d\) is modified position of the particle, \(n v_i^d\) is the particle with current velocity, \(n+1 v_i^d\) is the modified velocity, \(v_i^{pbest}\) is the velocity based on population best value, \(pbest_i\) and \(v_i^{gbest}\) is the velocity based on global best value, \(gbest_d\).

The velocity of the particle \(v_i^d\) and positions of the particle \(x_i^d\) are updated via the Equations given below, [19] [24]:

\[
n+1 v_i^d = \omega n v_i^d + c_1 r_1^d (pbest_i - n x_i^d) + c_2 r_2^d (gbest - n x_i^d)
\]

(25)

\[
n+1 x_i^d = n x_i^d + n+1 v_i^d
\]

(26)

\[
i = 1, 2, 3, \ldots, N, \quad d = 1, 2, 3, \ldots, D.
\]

where \(x_i = (x_i^1, x_i^2, x_i^3, \ldots, x_i^D)\) is the position of the \(i\)th particle, \(pbest_i = (pbest_1, pbest_2, \ldots, pbest_D)\) is the best local best position of a particle, \(gbest = (gbest_1, gbest_2, \ldots, gbest_D)\) is
the global best position discovered by the entire population, $v_i = (v_i^1, v_i^2, v_i^3, ..., v_i^D)$ is the velocity of a particle $i$, $c_1$ and $c_2$ are the acceleration constants. Here $n$ is the migration number, $r_1$ and $r_2$ are the random variables and $\omega$ is the inertia weight.

The modification of the standard PSO can be done by introducing a linearly time-varying acceleration constant in the evolutionary procedure as suggested in [23][24][25][26] and thus termed as MPSO. The MPSO modifies the acceleration constants $c_1$ and $c_2$ in Equation (25) with a high cognitive constant ($c_1$) and low social constant ($c_2$) at the start of the algorithm, and gradually $c_1$ is decreased and $c_2$ is increased to move the particle around the entire search space instead of converging toward a local minima. At the later stage of the optimization, the particles are allowed converge to the global optima.

$$c_1(\text{iter}) = (c_{1,\text{min}} - c_{1,\text{max}}) \frac{\text{iter}}{\text{iter}_{\text{max}}} + c_{1,\text{max}}$$

$$c_2(\text{iter}) = (c_{2,\text{max}} - c_{2,\text{min}}) \frac{\text{iter}}{\text{iter}_{\text{max}}} + c_{2,\text{min}}$$

Where $\text{iter}$ is the current iteration number and $\text{iter}_{\text{max}}$ is the maximum iteration number.

Then $a_{11}v_i^d$ and $a_{11}x_i^d$ should be under the constrained conditions as shown in Equation (29) and Equation (30).

$$n_{11}v_i^d = \begin{cases} n_{11}v_i^d, & -v_{\text{max}} \leq n_{11}v_i^d \leq v_{\text{max}} \\ v_{\text{max}}, & n_{11}v_i^d > v_{\text{max}} \\ -v_{\text{max}}, & n_{11}v_i^d < -v_{\text{max}} \end{cases}$$

$$n_{11}v_{\text{init}} = \begin{cases} n_{11}v_{\text{init}}, & n_{11}v_i^d > x_{\text{max}} \\ x_{\text{init}}, & n_{11}v_i^d < x_{\text{min}} \end{cases}$$

Where $v_{\text{max}}$ is the maximum value of $v$; $x_{\text{max}}$ and $x_{\text{min}}$ are the maximum and minimum value of $x$, respectively.

### 3 Methodology

#### 3.1 Fuzzy MF Parameters (a, b and c) Optimization Using MPSO

The design of fuzzy MF uses three parameters (a, b and c) that are subjected to the constraint; $0 \leq a < b < c \leq 255$ in accordance with the input brain image characteristics. These three fuzzy MF parameters are optimized using MPSO. Since MPSO uses objective function as entropy based on Equation (22) to find its optimal solution. Hence this optimization is considered as a minimization problem hence the fitness function is considered as inverse of objective function. The threshold is calculated from the optimal fuzzy MFs parameters and segmentation is carried out. The algorithm for obtaining the optimal threshold based on maximum entropy using MPSO is illustrated in Figure 3.

**Figure 3.** Algorithm for MPSO based fuzzy entropy MR Brain malignancy
The algorithm can be summarized as follows:
Initialization of the particle swarm for the position matrix \( X \) and the velocity matrix \( V \) are given below as:

\[
\begin{align*}
\text{n}\cdot x_{ij}^d &= x_{\text{min}} + (x_{\text{max}} - x_{\text{min}}) \cdot \text{rand( } ) \\
X &= \begin{bmatrix}
  x_{11} & x_{12} & x_{13} \\
  x_{21} & x_{22} & x_{23} \\
  \vdots & \vdots & \vdots \\
  x_{N1} & x_{N2} & x_{N3}
\end{bmatrix} \\
\text{n}\cdot v_{ij}^d &= -v_{\text{max}} + 2v_{\text{max}} \cdot \text{rand( } )
\end{align*}
\]

where, \( x_{\text{max}} \) and \( x_{\text{min}} \) are the maximum and minimum value of position \( (x) \) where \( x_{\text{max}} = L_{\text{max}} \), \( x_{\text{min}} = L_{\text{min}} + 1, L_{i2} - L_{i1} \geq 2 \), and \( L_{i1} - L_{i2} \geq 2 \); \( L_{\text{max}} \) and \( L_{\text{min}} \) are the corresponding maximum and minimum gray levels of the image. For each particle in the population, fitness value is calculated using the fuzzy entropy function \( H \). The comparison is carried out for the evaluated current fitness values with that of the fitness value of its previous best position. If the current fitness value is found to be better, then the previous best position is set as the current best position. Then comparison is carried out for the evaluated fitness value of each particle with the fitness value of the whole swarm’s previous best position, pbest. If the current value is better, consequently set the current position as the entire swarm’s previous best position. The velocity of each particle is updated using to Equation (25), simultaneously the position of each particle is updated with Equation (26), subject to constrain on Equations (29) and (30). The predefined maximum iterative time is the stopping criterion for the MPSO. If the terminating criterion is not satisfied, the MPSO will search for the next best particle in the swarm population. When the terminating criterion is attained, the threshold \( T \) is calculated based on the optimal fuzzy MF parameters \((a, b, c)\) then malignancy segmentation is processed.

4  Experimental Simulation Results

The entire simulation is carried out using MATLAB 7.10 on a Desktop with Intel® Core™ i5-Processor and 4GB RAM. The MPSO parameters were initialized as mentioned in Table I given below.

<table>
<thead>
<tr>
<th>MPSO simulation Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swarm size</td>
<td>25</td>
</tr>
<tr>
<td>Self-recognition coefficient, ( c_1 )</td>
<td>( c_{1 \text{ min}} ) 0.5, ( c_{1 \text{ max}} ) 2.5</td>
</tr>
<tr>
<td>Social coefficient , ( c_2 )</td>
<td>( c_{2 \text{ min}} ) 0.5, ( c_{2 \text{ max}} ) 2.5</td>
</tr>
<tr>
<td>Inertia weight, ( \omega )</td>
<td>1</td>
</tr>
<tr>
<td>Bird step</td>
<td>150</td>
</tr>
</tbody>
</table>

In substantiating this work, a set of malignant MR brain images are used for the simulation of the experimental data. Validation to check the effectiveness of the proposed method is done with existing methods such as, exhaustive search method and Otsu’s segmentation method [18]. The experimental simulation results of four malignant MR brain images are shown in figures.4-7. Each figure shows the test malignant MR brain image, the segmented images using the proposed method, exhaustive search method and Otsu segmentation. The test input MR Brain images used for segmentation include a malignancy in the left medial parietal cortex, shown in figure 4(a), in figure 5(a) the malignancy located in the supersellar region, in figure 6(a) the malignancy is located in the left occipito-parietal region and in figure 7(a) the malignancy is located in the medial parietal cortex. These input MR brain images were taken from MNI BrainWeb.
Figure 4(a) shows the test input image 1. Figure 4(b) shows the segmentation performed by the proposed method with an obtained optimal threshold value of $T=184.142$ and figure 4(c) shows the segmentation performed by the exhaustive search method with a threshold value of $T=184.142$. Figure 4(d) shows the segmentation performed by Otsu segmentation method with a threshold value of $T=74.2902$. From these figures it is observed that 4(b) and 4(c) shows the segmentation of the malignancy area in the MR brain more precisely, whereas the resulting performance of Otsu segmentation method is not in acceptable range. The comparison of the results for test image 1 is depicted in Table II.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Fuzzy MF (S function, Z function) parameters</th>
<th>Segmentation Threshold(T )</th>
<th>Fuzzy Entropy(H )</th>
<th>Time(sec )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Otsu Segmentation</td>
<td>NA</td>
<td>74.2902</td>
<td>4.8774</td>
<td>0.3273</td>
</tr>
<tr>
<td>Exhaustive search</td>
<td>(99, 203, 238)</td>
<td>184.142</td>
<td>7.4158</td>
<td>353.6871</td>
</tr>
<tr>
<td>MPSO</td>
<td>(99, 203, 238)</td>
<td>184.142</td>
<td>7.4158</td>
<td>2.0842</td>
</tr>
</tbody>
</table>

Table II shows the comparison of the results obtained using the three different methodology with respect to threshold, fuzzy entropy and computational time. Though Otsu segmentation method requires a very low computational time to segment, the fuzzy entropy obtained using this method is comparatively very low than other two methods. The exhaustive search method is one of the well proven conventional search methods to find the best value in entire search space of a problem. The exhaustive search method gives maximum fuzzy entropy of 7.4158; however the computational time required in finding the best values is very high. The proposed MPSO method gives the same threshold as that of exhaustive search method with minimum computational time which is 169 times lesser than that of exhaustive search method.
Figure 5(a) Test input MR image 2 (630 x 612 x 3), Figure 5(b) thresholding result by the proposed method for \( T=193.7421 \), Figure 5(c) results by exhaustive search method for \( T=193.7421 \), Figure 5(d) Otsu segmentation for \( T=82.3216 \)

Figure 5(a) shows the test input image 2. Figure 5(b) shows the segmentation performed by the proposed method with an obtained optimal segmentation threshold value of \( T=193.7421 \) and figure 5(c) shows the segmentation performed by the exhaustive search method for the input test image 2 with a threshold value of \( T=193.7421 \). Figure 5(d) shows the segmentation performed by Otsu segmentation method with a threshold value of \( T=82.3216 \). It is observed from figure 5(b), figure 5(c) and 5(d) that the segmentation of the malignancy area was performed more precisely in the proposed method. The comparison of the simulation results for the test image 3 is depicted in Table III.

Table III Comparison of results for test input image 2

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Fuzzy MF (S function, Z function) parameters</th>
<th>Segmentation Threshold(T)</th>
<th>Fuzzy Entropy(H)</th>
<th>Time(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Otsu Segmentation</td>
<td>NA</td>
<td>82.3216</td>
<td>4.9346</td>
<td>0.5149</td>
</tr>
<tr>
<td>Exhaustive search</td>
<td>(95, 225, 245)</td>
<td>193.7421</td>
<td>6.9207</td>
<td>358.0983</td>
</tr>
<tr>
<td>MPSO</td>
<td>(95, 225, 245)</td>
<td>193.7421</td>
<td>6.9207</td>
<td>4.7685</td>
</tr>
</tbody>
</table>

Table III shows the comparison of the results obtained using the three methods with respect to segmentation threshold, fuzzy entropy and computational time for the test input MR image 2. The performance of the proposed method gives the better results when compared with other two methods. The proposed MPSO method gives the same threshold as that of exhaustive search method with minimum computational time which is 73 times lesser than that of exhaustive search method.

Figure 6(a) Test image 2(300 x 300 x 3), Figure 6(b) thresholding result by the proposed method \( (T=222.6239) \), Figure 6(c) results by exhaustive search method \( (T=222.6239) \), Figure 6(d) Otsu segmentation\( (T=72.282) \)
Figure 6(a) shows the test input MR image 3. Figure 6(b) shows the segmentation performed by the proposed MPSO method with the obtained optimal threshold value of $T=222.6239$ and figure 6(c) shows the segmentation performed by the exhaustive search method with a threshold value of $T=222.6239$. Figure 6(d) shows the segmentation performed by Otsu segmentation method with a threshold value of $T=72.282$. From the output of malignancy in brain image segmentation revealed in the figure 6(b), figure 6(c) and figure 6(d), it is incidental that the malignancy area is more precisely segmented using the proposed method, whereas the performances of Otsu segmentation method was not acceptable. The comparison of the simulated test results for the test image 3 is depicted in table IV.

Table IV Comparison of results for the test input image 3

<table>
<thead>
<tr>
<th>Malignancy located in the left occipito-parietal region - Test image 3(300 x 300 x 3)</th>
<th>Methodology</th>
<th>Fuzzy MF (S function, Z function) parameters</th>
<th>Segmentation Threshold(T)</th>
<th>Fuzzy Entropy(H)</th>
<th>Time(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Otsu Segmentation</td>
<td>NA</td>
<td>72.282</td>
<td>5.9947</td>
<td>0.4194</td>
</tr>
<tr>
<td></td>
<td>Exhaustive search</td>
<td>(159, 247, 251)</td>
<td>222.6239</td>
<td>8.8191</td>
<td>354.286</td>
</tr>
<tr>
<td></td>
<td>MPSO</td>
<td>(159, 247, 251)</td>
<td>222.6239</td>
<td>8.8191</td>
<td>3.0286</td>
</tr>
</tbody>
</table>

Table IV shows the comparison of the results obtained using the three different methods with respect to threshold, fuzzy entropy and computational time for the test image 3. The comparison results prove that the proposed method outperformed the Otsu segmentation method with maximum fuzzy entropy with minimum computational time. The proposed MPSO method gives the same threshold as that of exhaustive search method with minimum computational time which is comparatively 62 times lesser than that of exhaustive search method.

Figure 7(a) Test image 4 (520 x 383), Figure 7(b) thresholding result by the proposed method ($T=225.3562$), Figure 7(c) results by exhaustive search method ($T=225.3562$), Figure7 (d) Otsu segmentation ($T=60.2353$)

Figure 7(a) shows the test input MR image 4. Figure 7(b) shows the segmentation performed by the proposed MPSO method with an optimal threshold value of $T=225.3562$ and figure 7(c) shows the segmentation performed by the exhaustive search method with a threshold value of $T=225.3562$. Figure 7(d) shows the segmentation performed by Otsu segmentation method with a threshold value of $T=60.2353$. From the experimentation results shown in figure 7(b), figure 7(c) and figure 7(d), the infected area is more precisely segmented using the proposed MPSO method. The comparison of the results for the test image 4 is depicted in table V.
Table V Comparison of results for the test input image 4

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Fuzzy MF(S function, Z function)parameters</th>
<th>Segmentation Threshold(T)</th>
<th>Fuzzy Entropy(H)</th>
<th>Time(se c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Otsu Segmentation</td>
<td>NA</td>
<td>60.2353</td>
<td>5.6704</td>
<td>0.4281</td>
</tr>
<tr>
<td>Exhaustive search</td>
<td>(155, 254, 255)</td>
<td>225.3562</td>
<td>6.9698</td>
<td>355.221</td>
</tr>
<tr>
<td>MPSO</td>
<td>(155, 254, 255)</td>
<td>225.3562</td>
<td>6.9698</td>
<td>3.9421</td>
</tr>
</tbody>
</table>

Table V shown above describes the comparison of the results obtained using the three methods with respect to threshold, entropy and computational time for the test image 4. It is inferred from the comparison table that the proposed method yields better segmentation than the other existing methods. The proposed MPSO method gives the same threshold as that of exhaustive search method with minimum computational time which is 90 times lesser than that of exhaustive search method. In all the test input MR brain images, the exhaustive search method obtains the maximum entropy; however the computational time required in finding the best values is very high. It is evident that the proposed method finds only the best fuzzy MF parameters; since the parameters obtained using both exhaustive search method and proposed method are similar for all test images.

The optimal fuzzy MF classification for the four test images are shown in following figures 8 - 11. These fuzzy membership function classification yields the maximum entropy for the corresponding test images.

Figure 8 The MF curves of the test image 1 with a =99, b =203, c =238 and maximal fuzzy entropy H =7.4158.

Figure 9 The MF curves of the test image 2 with a =95, b = 225, c = 245 and maximal fuzzy entropy H =6.9207.

Figure 10 The MF curves of the test image 3 with a =159, b =247, c = 251 and maximal fuzzy entropy H =8.8191.

Figure 11 The MF curves of the test image 4 with a =155, b =254, c = 255 and maximal fuzzy entropy H =6.9698.
Convergence Test

Due to the randomness of the proposed methodology, to show the frequency of convergence to the near optimal solutions, 25 trial runs were performed according to the parameter settings given in Table I. The convergence results are summarized in Table VI.

<table>
<thead>
<tr>
<th>Input Test Image</th>
<th>Fuzzy Membership Function Parameters for S-function &amp; Z-function</th>
<th>Fuzzy Entropy (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>Image 1</td>
<td>23</td>
<td>2</td>
</tr>
<tr>
<td>Image 2</td>
<td>24</td>
<td>1</td>
</tr>
<tr>
<td>Image 3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Image 4</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table VI: Convergence results with test input images for 25 trial runs

5 Conclusion

In this research paper, an optimal thresholding method based on bi-level MR brain image segmentation using maximum fuzzy entropy was introduced for better segmentation. The best optimal fuzzy MF parameters for setting the threshold value for segmentation were obtained using modified particle swarm optimisation. The modified PSO was introduced in to have effective exploration and exploitation. Analysis on the performances of the proposed method’s results was done with that of conventional adaptive thresholding method. The results show that the proposed method obtains acceptable performances in the segmentation simulation experiments conducted for different malignant test images. To confirm the optimized fuzzy parameters to be of global optimum, the results are compared against conventional search method (enumerative search method). The proposed MPSO method is capable of finding the global optimal fuzzy MF parameters as that of the conventional search method with minimum computational time. Validation of the results for the proposed method on consistency and robustness, convergence tests were carried out. It is evident from the results that more than 95% of the results are same as the conventional method. Hence, it is concluded that optimal maximum fuzzy entropy based image segmentation technique using MPSO is one of the effective methods for bi-level segmentation and can be adapted for segmenting the malignancy of the MR brain images effectively. The proposed MPSO method can also be extended to segment other sort of images for different applications in various areas of medical imaging. This method can be integrated as a substitute to bi-level segmentation to obtain better performances in any image processing applications.

References


