



MATHEMATICAL MODELLING OF FUZZY LOGIC APPLICATIONS

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ABSTRACT

The idea of modeling is to represent, manipulate and communicate original-world an object of normal life and that permits the simulation of complex processes, produce hypotheses and gives experiments. Many of the systems areas in biology, health, medicine, humanities and engineering are unwieldy with traditional analytical and mathematical methods, so in early 1990 it started to propose the concept of soft-computing or computational intelligence to refer to computational techniques and algorithms that exploit the tolerance for imprecision, uncertainty and partial truth. One such case is represented by fuzzy logic, a powerful tool for managing complex problems in a position where you have incomplete or not very accurate information. A review of the potential of fuzzy logic as a tool for mathematical modeling in life sciences is presented. Logic based on the two values True and false is sometimes inadequate when describing human reasoning. Fuzzy logic uses the whole interval between 0(false) & 1(true) except only two truth values to describe human reasoning. It should also be emphasized that both deterministic and stochastic models that have been developed to solve a verity of complex problems are characterized by mathematics based on binary logic.

Keywords: Fuzzy logic, mathematical modeling, Life sciences, biological systems, Transportation, Temperature.

1. INTRODUCTION

Mathematical Modeling has the aim of represent, manipulate and communicate real-world an object of daily life and that somehow allows the

simulation of complex processes, generate hypotheses and suggest experiments. In short, a good model should reflect the causal structure of the system under study, and be able to predict the outcome efficiently with new experiments [1]. However, there are many ways of observing an object or, equivalently, there are many different observers for the same object, so that the choice of the model (which is not unique) directly affects the interpretation and design; plus model selection are supported directly with the system under investigation, in which knowledge and information gained expert, structure, model assumptions, conceptual model and the required mathematical formalism, quality and type of experimental data is included (quantitative or qualitative), and the state of prior knowledge [2], [3]. Although one of the ultimate goals of classical mathematical modeling is to obtain expressions that allow quantitatively understand all the details and principles of biological systems, it is often difficult to perform [4]. Many of the systems areas in biology, medicine, humanities and engineering are unwieldy with traditional analytical and mathematical methods, so in early 1990, it proposed the concept of soft-computing or computational intelligence to refer to techniques and computational algorithms that exploit the tolerance for imprecision, partial truth and uncertainty for a specific problem, unlike hard-computing in which the accuracy and the whole truth is sought [5]. The methods of soft-computing can be considered derivatives of Machine Learning, which is as known to the branch of artificial intelligence that aims to develop techniques that allow a computer to generalize behavior from examples [6]. Many of the techniques of soft-computing often inspired by biology and includes models such as neural networks, fuzzy systems, Petri nets, Bayesian

networks, evolutionary algorithms, optimization ant colony, swarm intelligence, immune algorithms, among others [1], [5]. This requires designing architectures and radical models that

are conceptually different from traditional models, as well as the extraction of relevant information by data mining techniques [9].

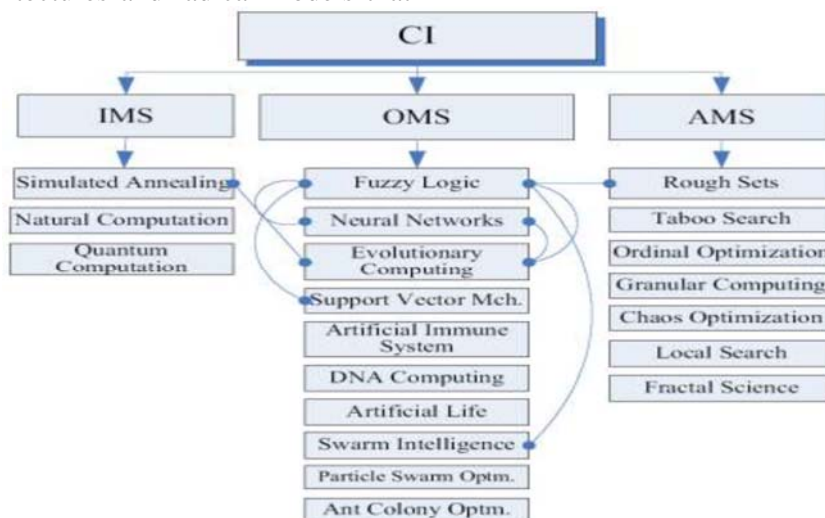


Figure 1. Classification of computational intelligence algorithms

2. FUZZY LOGIC CONTROLLER (FLC)

Fuzzy logic basically uses a logic and decision mechanism which doesn't have certain boundaries like human logic.

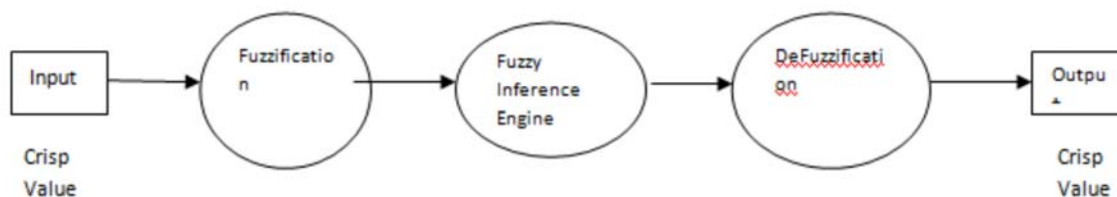


Figure 2. Decision mechanism

In the next step, the inference engine combines and maps fuzzy sets onto fuzzy set to get a fuzzy output set. During defuzzification one crisp value is chosen for the output variable by taking a weighted average of the various One of its most common implementation is in fuzzy logic based control mechanisms. Fuzzy logic control systems don't require complete model knowledge as other control system like PID. The fuzzy logic controller is easy to perform due to its simple control structure, ease of design and inexpensive cost. Main elements of Fuzzy logic controller are a fuzzication unit, a fuzzy logic reasoning unit, a knowledge base and a defuzzification unit. The fuzzy knowledge base contains 2 types of information: a data base defining the membership function of the fuzzy sets used as values for each system variable and a rule base which essential maps fuzzy values of the input to fuzzy values of the output. Input variables are often called crisp value. Then crisp value will be map into fuzzy sets to

corresponding fuzzy values in fuzzification, Inference engine consists of rule base and data base. The process of fuzzy inference involves

- I. Defining if then rules
- II. Defining membership functions
- III. Applying logical operations

3. FUZZY LOGIC APPLICATIONS

3.1. TRANSPORTATION

A) PROBLEM

Transportation problem is a wide human oriented field with diverse and challenging problems waiting to be solved. Characteristics and performances of transport systems-services, costs, infrastructures, vehicles and control systems are usually defined on the basis of quantitative evaluation of their main effects. Most of the transport decisions take place under imprecision, uncertainty and partial truth. Some objectives and constraints are often difficult to be measured by crisp values. Recently several authors observe that fuzzy logic is a useful

technique in real life transportation problems, starting from Pappis and Mamdani(1977)[11] who applied fuzzy logic to control an intersection of two one way streets or others used fuzzy logic in decision making system such as Gianluca(2012) and Brito et al. (2012).

B) TRANSPORTATION PLANNING

Trip generation constitutes the first stage in the traditional transportation planning process. Trip generation defines that how many people want to go out of their homes and what their purposes are. Researchers emphasized that more non-congestion trip generation models have better prediction and adaptation ability. Trip generation problem was solved using fuzzy logic by Kalic and Teodorovic (1997a) [8]. Fuzzy rule base was generated by learning from numerical examples. For this purpose, the procedure proposed by Wang and Mendel (1992a, b, c)[12] was used. Firstly, the available set of data was divided into two subsets: the first was used for generation of the fuzzy rule base, and the other was intended to be a control data subset. After the fuzzy rule base was created, the obtained fuzzy system was tested on both subsets of data. After the testing, the fuzzy logic approach proved to give the closest estimate of the actual number of trips generated in a given area.

C) TRIP DISTRIBUTION

Trip distribution constitutes the second stage in the traditional transportation planning process. Trip distribution models are used to

determine the number of trips between pairs of zones when the number of trips generated attracted by particular zones is known. Traffic flows and trip distribution resulted from human choices that are affected by social and individual variables of the commuters. A three phases fuzzy inference system (FIS) was proposed by Jassbi et al.(2011) [7] to map social and demographic variables to total number of trips between origin-destination (OD) pairs.

3.2. FUZZY LOGIC IN WASHING MACHINE

One of the most practical applications of FLS is using them as “process control system”. Washing machine is commonly used household appliances in India. On using a washing machine, the user manually sets the washing time, based on the type of clothes, , type of dirt and amount of clothes. Many people find it very difficult to decide that which cloth what need amount of washing time. Unfortunately, a precise mathematical relation between inputs and output cannot be defined. Building a washing machine with automatic washing time determination means building the following two subsystems

- I. The sensor system- collects data from outer environment and sends them to controller
- II. The controller system sets the washing time based on the information received from the sensor system a fuzzy logic controller will be used.

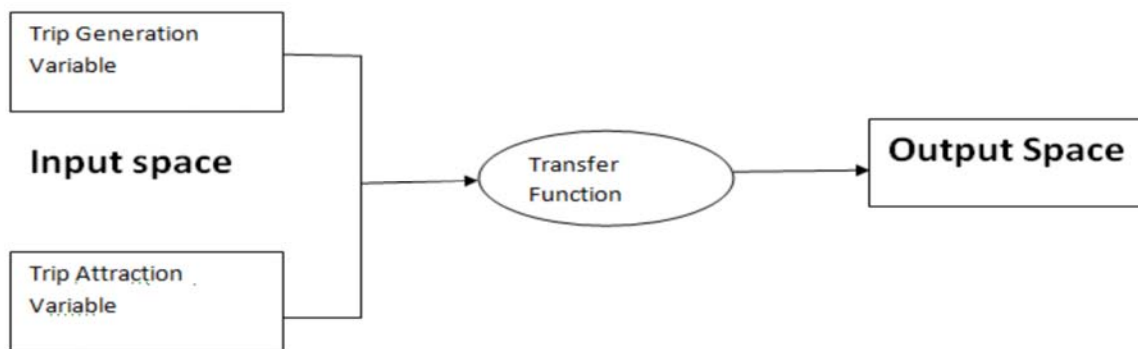


Figure 3. Conceptual model

The fuzzy logic controller for washing machine can consists of linguistic inputs i.e. type of dirt, type of clothes, dirtiness of clothes, mass of clothes. The sensors for all these linguistic inputs are also available. All the above LIs control the Linguistic output (LO) i.e. wash time, spin time, rinse time etc. Fuzzy logic controller mainly consists of three blocks i.e. fuzzifier, fuzzy

inference engine or fuzzy rule select and Defuzzifier. There is a membership function which turns the crisp input values into fuzzy values and after that suitable operation is performed on them. The process which converts crisp value into fuzzy value is known as fuzzification. The decision made by fuzzy logic controller and obtained from the rules known as

fuzzy rules. The result obtained from fuzzy inference engine is then processed to produce the output in crisp value and centroid method is used for defuzzification.

3.3. APPLICATION OF FUZZY LOGIC IN MEDICAL SCIENCE

Development of medical domain application has been one of the most active research areas. One example of the medical domain application is the detection system for heart disease. Fuzzy logic is used for this purpose clinical decision. Clinical decision support systems are widely categorized into two major groups namely (1) knowledge based CDSS and (2) non-knowledge based CDSS (Abbasi and Kashiyarndi, 2006). The knowledge based clinical decision support system comprises rules mostly in the form of IF–Then statements. Generally the data are associated with these rules. For instance, generate warning and more only if the pain intensity is up to a certain level. Generally the knowledge based CDSS encloses three main parts – knowledge base, inference rules and a mechanism to communicate. The adaptive guidelines from a knowledge base server prove to be much more effective than others in certain cases, such as that of chest pain management (Ali et al., 1999). Vagueness, impreciseness and uncertainty are the fundamental and indispensable aspects of knowledge, so as in several practical problems, the experts face vagueness in feature vectors and uncertainty in decision-making. Basically, a symptom is an uncertain indication of a phenomenon since it may or may not occur with it. Especially, uncertainty characterizes a relation between symptoms and phenomena (Straszecka, 2007; De et al., 2001). Sources of uncertainties may comprise that patients cannot describe accurately what has happened to them or how they suffer, doctors and nurses cannot explain exactly what they detect, laboratory reports' outcomes may be with some degrees of error, physiologists do not precisely understand how the human body works, medical researchers cannot precisely characterize how diseases modify the normal functioning of the body and no one can precisely determine one's prognosis (Szolovit, 1995; Kong et al., 2008). Decision support systems that are implemented with the support of artificial intelligence have the capability to espouse in a new environment and to learn with instance (Warren et al., 2000;

Anderson, 1997). In computer-aided support systems/expert systems, various methods are exploited to congregate information used for the process of decision-making. Statistical method, neural network, knowledge based methods, fuzzy logic rule-based, genetic algorithms and more are included in these methods (Abbasi and Kashiyarndi, 2006). Since the idea of computer-based CDSSs emerged at first, significant research has been made in both theoretical and practical areas. In recent years, clinical decision support system based on computer aided diagnosis methodologies have been proposed in the literature by which evaluating the data obtained by some of the methods or other sources (i.e., laboratory examinations, demographic and/or history data, etc.) from a computer-based application leads to a computer-aided diagnosis. The data analysis methods used in most of the proposed methods cannot provide clear and direct explanation for the decisions made to examine the risk factors for cardiovascular diseases as they are based on neural networks. Hence, a method based on easily obtained features capable of calculating the risk level of computer-aided diagnosis and providing explanation for the decisions made would be of immense clinical value (Tsipouras et al., 2008). So, the soft computing technique in particular the fuzzy logic technique is used for assessing the risk level of heart patients in developing the clinical decision support system of heart disease diagnosis. For better prediction of risk level, we make use of fuzzy logic, where the decision to be taken on heart disease of patients is based on the weighted fuzzy rules.

3.4. FUZZY LOGIC IN LIFE SCIENCES

A) Fuzzy logic

The concept of fuzzy logic or fuzzy sets was introduced in 1960 by Zadeh as a generalization of the theory of conventional sets [10]. Fuzzy logic is a type of logic with a series of values specified as a degree of truth instead of the true or false binary values [11]. Therefore, it is considered that the most important application of the fuzzy logic is in the managing of uncertainty [12]. Fuzzy logic is a powerful and suitable tool for handling complex problems in a position where there is incomplete or not very accurate information. In addition, the way that non-probabilistic uncertainties have been addressed with fuzzy logic have made the rise of the models with this paradigm has become popular in many

areas of knowledge, to the point of establishing related areas, occasionally identified as graphics diffuse, fuzzy interpolation fuzzy topology, fuzzy reasoning, fuzzy systems and fuzzy inference modeling [9]. According to Zadeh, the essence of fuzzy logic is that the exact reasoning is seen as a limiting case of approximate reasoning; also it proposes that everything is a matter of degree and that any system of logic can undergo the theory of fuzzy logic [10], [11]. This latter contributes to justify the features of fuzzy logic:

- ✓ It is conceptually easy to understand. Fuzzy logic is a more intuitive without the complexity of other far-reaching mathematical modeling method.
- ✓ It is flexible in accepting degrees and not only binary systems.
- ✓ It is tolerant of imprecise data.
- ✓ It is possible to model nonlinear functions of arbitrary complexity.

- ✓ It can build based on the experience of experts.
- ✓ It is based on the natural language of human communication.

Fuzzy sets can be considered as a generalization of the classical sets. Basically, a fuzzy set can have elements with a partial degree of membership and the membership degree is characterized by membership functions, which give flexibility fuzzy sets to model linguistic expressions employed daily. Operations are not based on the frequency of the phenomenon, but are rather based in terms of possibility that are qualitative and refer to capacities and potentialities. According to classical logic, an element can only be in one of the sets (exclusive), while according to the fuzzy logic, an element has different membership of the two sets lesser or greater extent (see Figure 4).

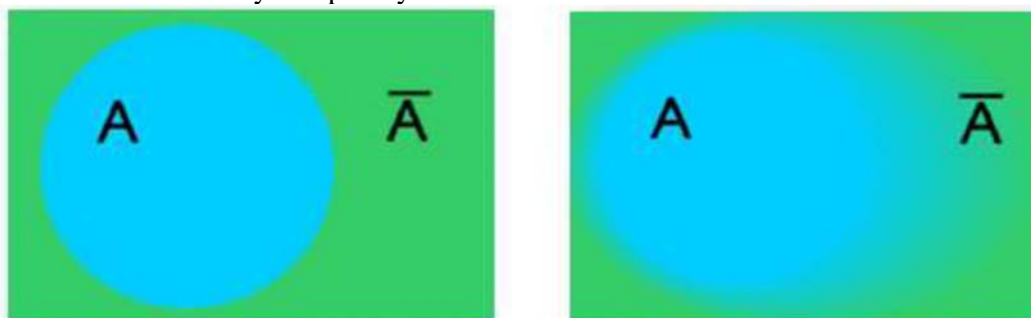


Figure 4. Sets in classical logic (left) and fuzzy logic (right).

More in detail, a classic set A can be expressed by the following membership function:

$$\mu_A(x) = \begin{cases} 1 & \text{si } x \in A \\ 0 & \text{si } x \notin A \end{cases}$$

In this case, the values to belong, $\mu_A(x)$ are only 0 and 1, depending on whether the variable x belongs or not to set A given. Meanwhile, a fuzzy set A in a universe of discourse X is defined by a membership function or membership $\mu_A(x)$ taking values in the interval $[0,1]$, and can be represented as a set of ordered pairs:

$$A = \{(x, \mu_A(x)) / x \in X\} A: X \rightarrow [0,1]$$

The membership values of $\mu_A(x)$ range from 0 (does not belong) and 1 (total membership) and where $\mu_A(x)$ is the function of belonging or membership.

B) Fuzzy mathematical models in life sciences

The fuzzy logic based systems have been used for models involving mining or oil exploration, communication systems, automation, fault diagnosis in nuclear electrical and power systems, engineering design, planning and transport, economy and finance, robotics, study and weather forecasting, aircraft design, flight simulators, search oilfields, study and prediction of tornadoes, assigning codes in mobile stations, bioinformatics problems, among

many others [5]. In the medical-biological area there are many systems in which qualitative data are included and have therefore been favored with the application of fuzzy logic models. Thus, the fuzzy logic method is suitable for biomedical applications because of the uncertain nature of medical and biological and relationships between these concepts; due to the complexity of the practice of medicine and biology, traditional approaches to quantitative analysis are not

appropriate in many cases. Some applications of fuzzy logic in the biomedical area include:

- ✓ Analysis of risk, diagnosis, prognosis, treatment and other medical decisions regarding cancer and other diseases.
- ✓ Analysis of medical images (magnetic resonance, smears, mammograms, X-rays) or biological (environmental samples, deforestation, rivers and oceans, species distribution, habitat studies and niches).
- ✓ Classification (separating benign from malignant lesions, animal sounds, anthropology).
- ✓ Show quantitative estimates of substances (drug use, distribution of oil in water, heavy metal contamination).
- ✓ Data analysis gene expression microarray.
- ✓ Modeling biological networks.

Moreover, the creation of hybrid mathematical models, inspired by different paradigms, is one of the tasks of data analysis has shown great response capacity. So, combine several methods of computational intelligence with other types of development of analysis is often given to give attention to the different levels and perspectives in solving problems, they can achieve very good results [8]. For example, about 70% of all studies covering problems of medical prediction uses neural networks, but produce a "black box" for the interpretation of the models. This can be solved using other tools and applications, being the combined with fuzzy logic one of the most popular. Because of this, it has increased the potential and interest of these hybrid models to explain real-life problems [9]. The use of fuzzy logic in biochemical models requires some numerical parameters in order to operate as initial values and coefficients of change, but the exact values of these numbers are not generally critical. Since the dynamic rules are defined in terms of fuzzy numbers, fuzzy logic models allow the calculation of the consequences of the dynamics of complex systems with imprecise variables. At the same time, fuzzy logic models are capable of representing complex systems to extremely high degree of accuracy when data are required [8].

Sokhansanj et al in 2004 showed a study in which they used fuzzy logic to analyze microarray data, identifying common patterns

between different models evaluated and achieving a better understanding of the system and its evolution. For these conclusions, they used an exhaustive search of cell cycle genes in yeast and assessed regulation at the transcriptional level, making the model was able to predict the behavior of the system, even after adjusting the model with a data set different. Another pioneering work on the use of fuzzy logic in the study of biological systems it is a model that analyzes the kinase pathway in TNF/EGF-induced insulin signaling in an article by Aldridge et al in 2009. The objective using fuzzy logic responds to the fact that it can encode probabilistic transitions and dynamics between network conditions in order to create simple representations and quite realistic of cellular signaling networks, which allowed a comprehensive analysis of the data, obtained experimentally. The construction of the fuzzy model was done by hand, because the authors wanted to test whether the fuzzy model could be adapted to test a priori knowledge and hypotheses with data to refine the understanding of the network and generate testable hypotheses.

4. CONCLUSION

Because knowledge of the various problems of study may limit the use of ODE models for mathematical approach, the use of modeling techniques soft computing, such as fuzzy logic, provides an opportunity to extract enough information to understand the system and make decisions despite not having accurate knowledge of the system. Particularly in biomedical sciences, the use of mathematical modeling with fuzzy sets provides the advantage that they are able to handle data in a non-discrete and manage uncertainty in the various stages of study. We have discussed various applications which were found to be very difficult to solve with traditional analytical techniques. Even hard computing models cannot deal effectively with these problems. In this situation fuzzy logic theory proved itself very beneficial and fruitful. There are so many problems like power system fault diagnosis, decision making behavior, crime investigation etc which can be handling by fuzzy logic.

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