REVIEW OF MATHEMATICAL MODELLING OF THIN LAYER DRYING PROCESS

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Abstract
Drying is an important technique to remove the moisture of the product. Various drying techniques are employed to dry different agricultural products. Each technique has its own advantage and limitation. So, choosing the right drying techniques is very important in the process of drying. The raw agricultural products with 80-90% moisture content are brought down to equilibrium moisture content for keeping in the short or long term storages. Present work involves the study of thin layer drying characteristics of different agricultural products under different drying process. The drying data fitted into different thin layer drying models. The performance of these models was investigated by comparing the coefficient of determination ($R^2$), reduced chi-square($\chi^2$) and root mean square error (RMSE) between the observed and predicted moisture ratio. On the basis of highest value of $R^2$ and lowest value of reduced chi-square($\chi^2$) and root mean square error (RMSE) appropriate model will be selected.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>a, b, c, g, h, n</td>
<td>Empirical constants in the drying models</td>
</tr>
<tr>
<td>k, k₀, k₁</td>
<td>Empirical constants in the drying models</td>
</tr>
<tr>
<td>n</td>
<td>Number constants</td>
</tr>
<tr>
<td>N</td>
<td>Number of observations</td>
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<tr>
<td>MR</td>
<td>Moisture ratio</td>
</tr>
<tr>
<td>MRexp</td>
<td>Experimental moisture ratio</td>
</tr>
<tr>
<td>MRpre</td>
<td>Predicted moisture ratio</td>
</tr>
<tr>
<td>M</td>
<td>Moisture content, (% dry basis)</td>
</tr>
<tr>
<td>Me</td>
<td>Equilibrium moisture content, (% dry basis)</td>
</tr>
<tr>
<td>Mo</td>
<td>Initial moisture content, (% dry basis)</td>
</tr>
<tr>
<td>Mt</td>
<td>Moisture content at t (% dry basis)</td>
</tr>
<tr>
<td>Mₜ+dt</td>
<td>Moisture content at t+dt (% dry basis)</td>
</tr>
<tr>
<td>t</td>
<td>Time, (min)</td>
</tr>
<tr>
<td>T</td>
<td>Temperature, (°C)</td>
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<tr>
<td>Tabs</td>
<td>Absolute temperature, (K)</td>
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Abbreviations:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>wb</td>
<td>Wet Basis</td>
</tr>
<tr>
<td>db</td>
<td>Dry basis</td>
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<tr>
<td>RMSE</td>
<td>Root mean square error</td>
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Introduction
Thin layer drying can be described as a drying of one layer sample particles or slices. Temperature distribution is considered uniform due to its thin layer characteristics. Lumped parameter model can be used as temperature is uniform in thin layer drying. The main mechanisms of drying are surface diffusion on the pore surfaces, liquid or vapor diffusion due to moisture concentration differences and capillary action in granular and porous foods due to surface forces. Generally, hygroscopic products dry in constant rate and subsequent falling rate periods and drying stops when equilibrium is established. During the constant rate period of drying, the physical form of the product and external conditions such as temperature, drying air velocity, direction of air flow and relative humidity have a great influence on the surface of the product being dried so called surface diffusion. When the surface film
of the solids or particles appears to be dried and the moisture content has been reduced to its critical moisture content then the first falling rate period begins. Unlike the constant rate periods, the falling rate period is controlled by liquid diffusion as a result of moisture concentration differences and the internal conditions of the product. The internal conditions such as moisture content, the temperature and the structure of the product play an important role in the falling rate periods. This phenomena is then replaced by the second falling rate periods of drying namely vapor diffusion due to moisture concentration difference and also internal conditions of the products. It has been accepted that the drying phenomenon of biological products during falling rate period is controlled by the mechanism of liquid and/or vapor diffusion [9].

2 Mathematical Models of Thin Layer Drying
Thin layer drying equations are important tools in mathematical modeling of drying.

Thin layer models

- Theoretical
- Semi-theoretical
- Empirical

Thin layer drying models that describe the drying phenomenon of biological materials mainly fall into three categories, theoretical, semi-theoretical and empirical.

The theoretical models explain the drying behaviors of the product clearly and can be used for all process conditions, but may include assumption about moisture mechanisms which may cause considerable error. The Fick’s second law equation is a theoretical model that has been used widely for thin layer drying process. Semi-theoretical models are generally derived from Fick’s second law and modifications of its simplified forms. The theoretical model takes into account only internal resistance to moisture transfer while the semi-theoretical and empirical models consider external resistance to moisture transfer resistance between product and air. A theoretical equation gives a better understanding of the transport processes but an empirical equation gives a better fit to the experimental data without any understanding of the transport processes involved.

The empirical models have similar characteristics with semi-theoretical layer drying. Commonly used semi-theoretical models in thin layer drying given in table 1.1.

Akpinar et al. [1] performed an experimental study and mathematical modeling of thin layer drying process of red pepper. The thin layer drying behavior of red pepper slices was experimentally investigated in a forced convective dryer consisting of fan, heaters, drying chamber and instrument for measurement. Drying experiments were conducted at inlet temperatures of drying air of 55, 60 and 70 °C and at a drying air velocity of 1.5 m/s. Initial moisture of the red peppers was 87.25% (wb), drying of the pepper continued until no further changes in their mass were observed. Final moisture content of the red pepper obtained was about 10% (wb). The times to reach 10% moisture content from the initial moisture content at the various drying air temperatures were found to be between 160 and 300 min. According to the results of the multiple linear regression analysis, among the 11 thin layer-drying models, the approximation of the diffusion model could adequately describe the thin layer drying behavior of red peppers. Constant drying rate period was not observed under any of the test conditions of this investigation, the red pepper drying process occurring in the falling rate.

Madamba et al. [2] studied the thin-layer drying characteristics of Garlic Slices. The thin-layer drying characteristics of garlic slices (2-4 mm) were investigated for a temperature range 50-90°C, a relative humidity range 8-24%, and an airflow range 0.5-1 m/s. Initial moisture content by garlic slice varied from 60-64% wet basis and was determined in a vacuum oven under certain condition. The effects of temperature, thickness, RH and air velocity were analyzed using analysis of variance (ANOVA), and this clearly showed that temperature and slice thickness were significant factors in drying. In this experiment, it was concluded that most of the drying of garlic slices takes place in the falling rate period. Four mathematical models available in the literature were fitted to the experimental data, with the Page and the two-compartment models giving better predictions than the single-term exponential and Thompson’s model.
Table 1.1 semi-theoretical models of thin layer drying

<table>
<thead>
<tr>
<th>Model no.</th>
<th>Model name</th>
<th>Model</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Newton</td>
<td>MR=exp(-kt)</td>
<td>Mujumdar [25]</td>
</tr>
<tr>
<td>2</td>
<td>Page</td>
<td>MR=exp(-kt^n)</td>
<td>Diamante and Munro [26]</td>
</tr>
<tr>
<td>3</td>
<td>Modified page</td>
<td>MR=exp[-(kt)^n]</td>
<td>White et al. [27]</td>
</tr>
<tr>
<td>4</td>
<td>Henderson and Pabis</td>
<td>MR=a exp(-kt)</td>
<td>Zhang and Litchfield [28]</td>
</tr>
<tr>
<td>5</td>
<td>Logarithmic</td>
<td>MR=a exp(-kt)+c</td>
<td>Yagcioglu et al. [29]</td>
</tr>
<tr>
<td>6</td>
<td>Two term</td>
<td>MR=a exp(ko)+b exp(-kt)</td>
<td>Henderson [30]</td>
</tr>
<tr>
<td>7</td>
<td>Wang and Singh</td>
<td>MR=1+a exp(-bt^2)</td>
<td>Wang and Singh [19]</td>
</tr>
<tr>
<td>8</td>
<td>Approximation of diffusion</td>
<td>MR=a exp(-kt)+(1-a)exp(-ktb)</td>
<td>Yaldiz and Ertekin [15]</td>
</tr>
<tr>
<td>9</td>
<td>Verma et al.</td>
<td>MR=a exp(-kt)+(1-a)exp(-gt)</td>
<td>Verma et al. [31]</td>
</tr>
<tr>
<td>10</td>
<td>Modified Henderson</td>
<td>MR=a exp(-kt)+b exp(-gt)+c exp(-ht)</td>
<td>Karathanos [32]</td>
</tr>
<tr>
<td></td>
<td>and Pabis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Two term exponential</td>
<td>MR=a exp(-kt)+(1-a)exp(-kt)</td>
<td>Shara et al. [33]</td>
</tr>
</tbody>
</table>

Rafiee et al. [3] studied thin layer drying properties of soyabean. Experiments were conducted at inlet drying air temperatures of 30, 40, 50, 60 and 70 °C and at a fixed drying air velocity of 1 m/s. The thin layer drying behavior of soyabean was experimentally investigated and the mathematical modeling performed by using thin layer drying models provided in the literature. The experimental moisture ratio was fitted to 13 thin layer drying models. Midilli [4] model gave the lowest RMSE and the highest regression coefficient values. Hence, the Midilli model was chosen to represent the thin layer drying of soybeans.

Akpinar [5] investigated suitable thin layer drying curve model for some vegetables and fruits. This study presents a mathematical modeling of thin layer drying of potato, apple and pumpkin slices in a convective cyclone dryer. He performed experiment in convective cyclone dryer. The dryer consists of fan, resistance and heating control system, air duct, drying chamber in cyclone type and measurement instruments. In which Potato slices of 83% (wb), apple slices of 87% (wb), pumpkin slices of 93% (wb) average initial moisture content were dried to 10% (wb), 13% (wb), 6% (wb), respectively, at temperatures of 60, 70 and 80 °C in the velocities of drying air of 1 and 1.5 m/s. It was noticed that the Midilli–Kucuk model gave the highest correlation...
coefficient ‘r’ and the lowest chi-square (X²) for all drying conditions.

Rao et al. [6] performed study of drying characteristics of large-cardamom of Sikkim. The moisture content of freshly harvested cardamom capsules is 70 - 80% (on wet basis). The moisture content is reduced to less than 10% for preservation by drying using traditional method and improved flue pipe curing house. In the traditional method, the cardamom capsules are dried in a locally made curing house known as bhatti. In this method, the cardamom capsules are spread on bamboo or wire-mesh and are exposed directly to smoke produced by burning wooden logs resulting in deposition of shoots on the surface of the dry cardamom. The thermal efficiency of the traditional drying system is very low about 5-15% only. To improve the quality of the dry cardamom, the bhatti system was modified. In the modified system, the combustion gas flows through a bundle of tubes, and air is heated up while passing over the surface of the tubes. The cardamom capsules are dried by exposing it to hot air instead of smoke resulting in good quality products. The required temperature of air for drying cardamom is in the range of 55 to 60 °C.

Modeling of thin-layer drying of potato slices in length of continuous band dryer done by Aghbashlo et al. [7]. They studied thin-layer drying behavior of potato slices in a semi industrial continuous band dryer. Potato slices with thickness of 5 mm were used for drying experiments. The experiments were done at air temperatures of 50, 60 and 70 °C, air velocities of 0.5, 1 and 1.5 m/s and chain linear velocities of 1.85×10⁻⁴, 2.22×10⁻⁴ and 2.78×10⁻⁴ m/s. The initial moisture content of potato slices was observed to be 3.62 ± 0.12 (kg water/kg dry matter). The equilibrium moisture content of potato slices varied from 0.121 ± 0.023 to 0.011 ± 0.007 (kg water/kg dry matter). The moisture ratio of the samples decreased continually with axial position. There was a rapid moisture removal from the products at the initial stages of drying process which later decreased with progressing in length of dryer. Three drying models were fitted to the experimental data of moisture ratio in order to assess a suitable form of the drying curve for potato drying. The Page model was selected as the best according to R², X² and RMSE.

Garavand et al. [8] determined the mathematical modeling of thin layer drying kinetics of tomato. Thin-layer drying kinetics of tomato was experimentally investigated in a pilot scale convective dryer. Experiments were performed at air temperatures of 40, 60, and 80 °C and at three relative humidity of 20%, 40% and 60% and constant air velocity of 2 m/s. The drying behavior of tomato slices in a pilot dryer was investigated at three different drying air temperatures and three different drying air relative humidities. Initially moisture content of tomato was 93.5 % (wb), the drying process was stopped after no further change in weights was observed. After drying final moisture content was 15 % (wb). The times to reach equilibrium moisture (15%) from the initial moisture content at three temperatures and air relative humidity were found to be between 420 and 1800 min. The high values of coefficient of determination and the low values of reduced sum square errors and root mean square error indicated that the Midilli model could satisfactorily illustrate the drying curve of tomato.

Panchariya et al.[9] investigated thin-layer modeling of black tea drying process. The tea manufacturing process involves several steps: withering, rolling or cutting tearing and curling (CTC), fermentation, drying, and sorting. The withering and drying processes are basically moisture removal process although some chemical changes occur during these two processes. An experimental dryer was developed for determining the kinetics of black tea drying. Drying characteristics of tea were examine during heated ambient air for the temperature range 80–120°C and air flow velocity range 0.25–0.65 m/s. The data of sample weight, dry-bulb and wet-bulb temperatures and air velocity of the drying air were recorded continuously during each test. The dryer consisted of three basic sections: air flow control section, heating control section, and sample platform. The air flow was circulated in the dryer by a centrifugal fan, driven by a 1.5 kW, three-phase electric motor. The Lewis model gave better predictions than other models, and satisfactorily described the thin-layer drying characteristics of black tea particles.

Meisamiand etal. [10] Studied kinetics of thin layer drying of apple. Mathematical models of thin-layer drying of apple were studied and verified with experimental data. The thin-layer drying kinetics of apple slices was
experimentally investigated in a laboratory by a convective dryer. Experiments were performed at air temperature between 40 and 80 °C, velocity of 0.5, 1 and 2 m/s, and thickness of thin layer of 2, 4, 6 mm. Besides the effects of drying air temperature and velocity, effects of slice thickness on the drying characteristics and drying time were also determined. The initial moisture content of apples was obtained as 5.0-6.4 % (db), moisture content approached near zero after drying. Drying curves obtained from the experimental data were fitted to the thin layer drying models. The results showed that Midilli model was the best model for describing the drying curves of apples.

Ertekin et al. [11] observed the thin layer drying behavior of eggplants in a laboratory dryer consisting of electrical fan, heating unit and drying chamber. Drying characteristics of eggplants were determined using heated ambient air at temperatures from 30 to 70 °C and velocities from 0.5 to 2.0 m/s. Drying curves obtained from the experimental data were then fitted to the different semi-theoretical and/or empirical thin layer drying models. The Midilli model was found to be the best model for describing the drying curves of eggplants.

Madamba [12] performed thin layer convection drying on osmotically pre-dried young coconut, strips, both thin and thick. A drying air temperature range of 50–70°C and an air flow of 0.25m/s was used to dry samples soaked in three sugar solution concentrations during the osmotic drying phase, with the convection drying. Drying air temperature and slice thickness were significant factors during the hot air drying stage. Sugar solution concentration did not significantly affect the drying rate of coconut strips during convection drying. The thin layer drying characteristics during hot air drying of coco-nut strips were satisfactorily described by the Page model.

Akpinar et al. [13] performed an experimental study and mathematical modeling of thin layer drying process of long green pepper in solar dryer and under open sun. An indirect forced convection solar dryer consisting of a solar air collector and drying cabinet was used in the experiments. Natural sun drying experiments were conducted for comparison at the same time. During the drying experiments the temperature of ambient air ranged from 23 to 45.3°C, the temperature of drying air at the inlet of the drying cabinet from 43.9 to 64.8°C and the temperature of drying air at the outlet of the drying cabinet from 33.6 to 57.7°C. Direct instantaneous solar radiation was 971W/m². Wind speeds varied between 0 m/s and 3.2 m/s during the days of the experimental work. The long green peppers of 4g water/g dry matter average initial moisture content were dried to 0.10 g- water/g dry matter in the solar drying. Among the models, the logarithmic model for forced solar drying of green pepper were found best to explain the thin layer drying behavior of long green peppers.

Zaman et al. [14] studied thin layer solar drying of rough rice. This paper presents a set of simple empirical equations for natural air flow solar drying of rough rice in mixed-mode type dryer, box-type dryer and open floor drying system. They examined all three dryers and results were evaluated to compare them. The air flows in the dryers were induced by the temperature difference between the air temperatures inside and outside the dryers. Total drying time required to dry rough rice from 31.6% (d.b) to 16.3% (d.b) was 5 to 8 hours in mixed mode dryer , 6 to 11 hours in box dryer and 7 to 12 hours on open floor.

Yaldiz [15] obtained Mathematical modeling of thin layer solar drying of sultana grapes. An indirect forced convection solar dryer consisting of a solar air heater and a drying cabinet was used in the experiments. During the drying experiments, ambient air temperature ranged from 32.15 to 33.32 °C, ambient air relative humidity from 66.43 to 75.76%, drying air temperature from 32.40 to 40.30°C, drying air relative humidity from 57.73 to 75.11% and solar radiation from 790.3 to 802.0 W/m². The initial moisture content of the grapes ranged from 2.6 to 3.3 kg water per kg dry matter and was reduced to the final moisture content of 0.16 kg water per kg dry matter. Eight different thin layer mathematical drying models were compared according to their coefficient of determination to estimate solar drying curves. According to the results, the two-term drying models could adequately describe the thin layer solar drying behavior of Sultana grapes.

Prasad et al. [16] studied drying characteristics of ginger by drying ginger in
direct type natural convection solar biomass (hybrid) dryer and under open sun. By the experiment they observed that ginger dried at less time in the hybrid solar drier than the open sun drying. The overall efficiency of the dryer was 13% to 18% under winter and summer climatic condition respectively. Average drying air temperature of 60 °C with average air velocity of 0.6m/s was sufficient for drying ginger. Open sun dried products were dark, while solar-biomass dried products were light. They evaluated the quality of ginger in the different dryer based on color values. The modified Page model was the most effective model to describe the mass transport behavior of ginger during air drying.

The drying characteristics for chili pepper using sun and solar drying were investigated by Akintunde [17]. Pretreated and untreated chili pepper were used. It was found that pretreated chili pepper dried faster than untreated chili pepper while the drying of both samples occurred in falling rate period. Chili pepper of uniform size was used for study. The drying time for solar dried samples which varied from 35 to 50 h was lower than that for sun dried samples which varied from 45 to 70 h. This is similar to the results obtained during the drying of chili in a solar dryer in which moisture content of red chili reduced to 0.05 kg/kg (db) from 2.85 kg/kg (db) in 20 h of drying while it took 32 h to bring down the moisture content of okra to 0.40 kg/kg (db) by sun drying method. In another study it was observed that open sun drying of red chili took 150 and 102 h compared to a green house type solar drier which took 90 and 66 h, while a solar cabinet drier took 54 and 36 h to reduce moisture content from 300% to 9% (db) for un punched and punched chilies, respectively. The Page model, which gave higher values of coefficient of determination and lower values of reduced chi-square, MBE and RMSE was considered the best for predicting the drying characteristics of sun and solar dried chili pepper.

Yaldiz et al. [18] investigated the drying characteristics of green bean and okra. The drying study showed that the times taken for drying of green bean and okra from the initial moisture contents of 89.5% and 88.7% (wb) to final moisture content of around 15±0.5% (wb) were 60 and 100 h in open sun drying, respectively. The constant rate period was absent in drying curves. The drying process took place in the falling rate period. The drying data were fitted to thirteen thin-layer drying models. The drying process occurred in falling rate period, and no constant rate period of drying was observed. Estimations by Approximation of diffusion (for green bean) and Midilli models (for okra) were in good agreement with the experimental data obtained.

Hossain et al. [19] presented the design of the solar dryer, fabricated and installed at Spices Research Centre, Bogra for drying of chilli. A DC fan of 10 watt was used for exhausting moisture with the help of a solar panel of 15 watt. In the solar dryer 8.75 kg dried chilli was obtained from 30 kg of red ripe chilli. The final drying levels of the red chili were obtained after 41 h at upper tray and 46 h at lower tray but took about 91 h in the open sun drying system, having the same weather condition. The area of collector designed for 7.5 m² dryer was 4 m². The necessary volumetric air flow rate was calculated as 0.11 m³/s at vent area of 0.03 m².

Banout et al. [20] performances of a new designed Double-pass solar drier (DPSD) were compared with those of a typical cabinet drier (CD) and a traditional open-air sun drying for drying of red chilli in central Vietnam. The drying times (including nights) to reach the desired moisture content of 10% (on a wet basis) were 32 and 73 h respectively. During open-air sun drying the desired moisture content of 10% (on a wet basis) could not be reached even after 93 h of drying (including nights). The overall drying efficiencies of DPSD and CD to reach the desired moisture content of 10% (on a wet basis) were 24.04% and 11.52% respectively while the overall drying efficiency of open-air sun drying to reach the desired moisture content of 15% (on a wet basis) was 8.03%.

Hossain et al. [21] explained about solar drier in which the collector and drying chamber were made of plain metal sheets and wooden frames in a number of small sections and were joined together in series. These sections can be opened easily for transportation from one place to another. Glass wool was used between the two metal sheets at the bottom of the drier as an insulation material to reduce the heat loss from the bottom of drier. The collector was painted
black to facilitate absorption of solar radiation. The drying area of the drier unit was same as that of the collector. Both the collector and the drying units were covered by 0.2 mm thick transparent UV stabilized plastic sheet. The plastic sheet was fixed on the collector side of the drier to the metal frame using U-type aluminum channel and rubber rope. At the drying unit one end of plastic sheet was fixed to a metal tube, which allows rolling of the plastic sheet up and down for loading and unloading of the drier.

3 Findings of the review

<table>
<thead>
<tr>
<th>Product</th>
<th>Drying Process</th>
<th>Process Condition</th>
<th>Best model</th>
<th>Reference</th>
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<tbody>
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<td>Apple(slice)</td>
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<td>60-80</td>
<td>1-1.5</td>
<td>Midilli</td>
</tr>
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<td>0.5-1.5</td>
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<td>1.5</td>
<td>Diffusion</td>
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<td>0.1-0.35</td>
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<td>2</td>
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<td>0.5-1.5</td>
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<td>Solar</td>
<td>60</td>
<td>0.6</td>
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<td>50-70</td>
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<td>0.5-2.0</td>
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</tbody>
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4.

3 Mathematical modeling approach:
Mathematical modeling of the sample has to be done on the basis of drying characteristics obtained from the experimental investigation. Thin layer drying requires some of the important parameters to get the drying characteristics of the product.

4.3 Important parameters:
4.3.1 Moisture content
The quantity of moisture present in a material can be expressed either on the wet basis or dry basis and expressed either as decimal or percentage. The moisture content on the wet basis is the weight of moisture present in a product per unit weight of the undried material, represented as,

\[ M_{wb} = \frac{W_0 - W_d}{W_0} \]  

\[ \text{eq.(1)} \]

While the moisture content on the dry basis is the weight of moisture present in the product per unit weight of dry matter in the product and represented as,

\[ M_{db} = \frac{W_0 - W_d}{W_d} \]  

\[ \text{eq.(2)} \]

Percentage

\[ M_{wb} = M_{wb} \times 100; \]  

\[ \text{eq.(3)} \]

Percentage

\[ M_{db} = M_{db} \times 100; \]  

\[ \text{eq.(4)} \]

The moisture contents on the wet and dry basis are inter-related according to the following equations,

\[ M_{wb} = 1 - \left( \frac{1}{(M_{db} + 1)} \right) \]  

\[ \text{eq.(5)} \]

The moisture content on the wet basis is used normally for commercial purposes, while the moisture content on the dry basis has tended to be employed for engineering research designation, because the weight change associated with each percentage point of moisture reduction on the dry basis is constant as against the wet basis where the amount of water involved in a moisture content reduction of one percent changes as drying progresses, because the weight of water and total crop weight change.

4.3.1.2 Equilibrium moisture content (Me)
A crop has a characteristic water vapor pressure at a particular temperature and moisture content. The equilibrium moisture content is the moisture content at which the product is neither gaining nor losing moisture. It is a dynamic equilibrium which changes with relative humidity and temperature.

4.3.1.3 Moisture ratio (MR):
Moisture ratio is one of the important criteria to determine the drying characteristics of agricultural product. MR can be determined according to external conditions. If the relative humidity of the drying air is constant during the drying process, then the moisture equilibrium is constant too. In this respect, MR is determined as in Eq.

\[ MR = \frac{M_t - M_e}{M_o - M_e} \]  

\[ \text{eq. (6)} \]

If the relative humidity of the drying air continuously fluctuates, then the moisture equilibrium continuously varies so MR is determined as in Eq.(7) given by Diamante and Munro[26]

\[ MR = \frac{M_t}{M_o} \]  

\[ \text{eq. (7)} \]

4.3.2 Drying rates:
Agricultural products differ from most other materials dried frequently, such as textiles in a Laundry, sand, stone, dust or paper. Agricultural products (which are hygroscopic) has always some residual moisture after the drying while for non-hygroscopic material drying continued up to zero moisture content. Because of hygroscopic products moisture is trapped in closed capillaries. The rate of moisture flow is only approximately proportional to its vapor pressure
difference with the environment because of the crop resistance to moisture flow. There are two main drying rate regimes for agricultural products, namely the constant drying rate period and the falling drying rate period.

\[
\text{Drying rate} = \frac{M_{t+dt} - M_t}{dt}
\]

**eq.(8)**

4.3.2.1 Constant drying rate period

During the constant drying rate period (see Fig. 4.2), drying takes place from the surface of the product and is simply the evaporation of moisture from the free-water surface. The rate of moisture removal during this period is mainly dependent on the surrounding conditions and only affected slightly by the nature of the product. During this period, the product surface is saturated with moisture with its temperature fairly constant and approximately equal to the wet bulb temperature. The end of the constant drying rate period is marked by a decrease in the rate of moisture migration from within the product below that sufficient to replenish the moisture being evaporated from the surface. At this stage, which defines the critical moisture content (see Fig. 4.2), environmental conditions cease to play much role in the rate of drying. For non-hygroscopic materials, all drying takes place within the constant drying rate regime.

Environmental factors, namely the vapor pressure difference between the drying air and the wet surface, the surface area of the product exposed to the drying air, the mass transfer coefficient and the drying air velocity, are related to the drying.

Fig. 4.2 Schematic illustration of drying rate periods: 1-2 the heating period (constant moisture content); 2-3 the constant rate drying period; 3 the critical moisture content; 3-4 the first falling rate period; 4-5 the second falling rate period.

4.3.2.2 Falling drying rate period

The critical moisture content of the product is the minimum moisture content at which the minimum rate of free moisture migration from within the product to the surface equals the maximum rate of moisture evaporation from the surface. Below the critical moisture content is the falling drying rate period (see Fig. 4.2). This drying rate regime is dependent essentially on the rate of diffusion of moisture from within the product to the surface and also on moisture removal from the surface. It is subdivided usually into two stages, namely.

1. The first falling drying rate period which involves the unsaturated surface drying; and
2. The second falling drying rate period where the rate of moisture diffusion to the surface is slow and is the determining factor.

For agricultural products, the duration of each of these drying regimes depends on the initial moisture content and the safe storage moisture content. For grains, the initial moisture content is usually below the critical moisture content, thus all drying takes place within the falling rate regime. However, for fruits, most vegetables and most tropical tuber crops, the initial moisture content is usually above the critical moisture content, thus the drying of these products would take place within both the constant and falling rate periods. Both the external factors and internal mechanisms
controlling the drying processes in the two main rate regimes are important in determining the overall drying rate of products.

### 4.4 Determination of appropriate model:

Mathematical modeling of the drying of food products often requires the statistical methods of regression and correlation analysis. Linear and nonlinear regression analyses are important tools to find the relationship between different variables, especially, for which no established empirical relationship exists. Thin layer drying equations require MR variation versus time ‘t’. Therefore, MR data plotted with time t and regression analysis is performed with the selected models to determine the constant values that supply the best appropriateness of models. The validation of models can be checked with different statistical methods.

The most widely used method is performing correlation analysis (r), reduced chi-square ($\chi^2$) test and root mean square error (RMSE) analysis.

#### Correlation coefficient (r)-

\[
\rho = \frac{\sum_{i=1}^{N} \text{MR}_{\text{pre},i} \cdot \text{MR}_{\text{exp},i} - \sum_{i=1}^{N} \text{MR}_{\text{pre},i} \cdot \sum_{i=1}^{N} \text{MR}_{\text{exp},i}}{\sqrt{\left( N \sum_{i=1}^{N} (\text{MR}_{\text{pre},i})^2 - (\sum_{i=1}^{N} \text{MR}_{\text{pre},i})^2 \right) \left( N \sum_{i=1}^{N} \text{MR}_{\text{exp},i} - (\sum_{i=1}^{N} \text{MR}_{\text{exp},i})^2 \right)}}
\]

\[\text{eq.(9)}\]

#### Reduced chi-square ($\chi^2$) –

\[
\chi^2 = \frac{\sum_{i=1}^{n} (\text{MR}_{\text{exp},i} - \text{MR}_{\text{pre},i})^2}{N-n}
\]

\[\text{eq. (10)}\]

#### Root mean square error (RMSE)-

\[
\text{RMSE} = \left[ \sum_{i=1}^{N} \frac{1}{N} (\text{MR}_{\text{exp},i} - \text{MR}_{\text{pre},i})^2 \right]^{1/2}
\]

\[\text{eq. (11)}\]

The highest correlation coefficient (r) and the lowest chi square ($\chi^2$) and root mean square error (RMSE) values are required to select the best suitable model to explain the thin drying process. Once the drying curves obtained from the experimental data, then it will be fitted to the different semi-theoretical thin layer drying models (shown in table 1.1). The model which satisfies these requirements will be selected to represent the thin layer behavior of the product.

### 5 Conclusions

A comprehensive review of the fundamental principles and theories required for the mathematical modeling of thin layer drying has been presented. For different agricultural product different thin layer model was selected as per the statistical approach.

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