BIANCHI TYPE-V MODEL COUPLED WITH STRINGS IN *F(R)* THEORY OF GRAVITATION IN PRESENCE OF STRANGE QUARK MATTER

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Abstract: In this paper, we investigated the anisotropic Bianchi type-V model in the presence of strange quark matter with the appearance and non-appearance of strings in *f(R)* gravitation. In this case, R is the Ricci scalar of space-time. Consider the following condition to achieve an exact solution to the field equation. i)hybrid scale factor, ii)proportionality of shear scalar(σ) with expansion scalar(Θ), and iii)power-law relationship between scalar field *F(R)* and average scale factor *a(t)*.We researched some physical and geometrical features of the models and studied their behaviour using graphical representation.

Keywords:Bianchi type-V model; strange quark matter;cosmic strings; *f(R)* gravity;Hybrid scale factor; Deceleration parameter;De-SitterExapansion.

1. Introduction

The theory of general relativity is one of the most important theory in the history of physics. It unifies the description of gravity as a geometric characteristics of space & time by generalizing special relativity & Newton's laws of universal gravitation. The recent observational observations shows that currently our universe is undergoing to accelerating expansions. This observations are evident by recent observations[1], [2], [3], [4]. To get the additional information & behavior of universe, we modify Einstein theory of gravitation. By use of Einstein-Hilbert action with Einstein field equations, we can derive the modified theory of gravitations like *f(R)* gravity, *f(T)* gravity, *f(R,T)* gravity & *f(G)*(where *R* is the Ricci scalar, *T* is the torsion scalar & *G* is the Gauss-Bonnet scalar).

Among all these theories, The*f(R)* theory of gravity has received more attention in the recent years and also explained by substitution of *f(R)* at the place of Ricci scalar R in the Einstein-Hilbert action. The *f(R)* theory of gravity is a very simple combination of early time inflation & acceleration of late time. The model in this theory consist of higher order curvature invariant as functions of the Ricci scalar R.Nojiri&Odinstov[5] discussed modified gravity with negative & positive power of Ricci curvature as a function of f(R) and describe the inflation & cosmic acceleration.A.S.Eddington[6] have solved the various action integrals for f(R) theory of gravity. Capozziello&Laurentis studied future prospective of extended theory of gravity by different approaches [7]. Capozziello&Franaviglia[8]gives extended theory of gravity with their cosmological & astrophysical application. H.Weyl[9] discussed new extension of theory of relativity.R.Ferraro[10] studied f(R) & f(T) theories of modified gravity in the sense of metric & palatine formulations.Snthi*et.al* [11],Godani&samanta[12], [13],Aditya *et.al.*[14],Samanta&Godani[15], [16], [17], Shamir[18], [19] , Sharif &Kausar[20]Amir &Sattar[21] , Sharif & Samir[22] are the researchers who done the study different cosmological models in *f(R)* theory of gravity.

M.Farasat& Shamir studied different Bianchi type cosmological models within a framework of*f(R)* gravity. M.Sharif&H.RizwanaKausar[23] devoted to study of Bianchi type-III model and discussed anisotropy behavior of fluid in *f(R)* gravity. M.sharif&M.Farasat Shamir[24] obtained the exact singular & non-singular solution of

Bianchi type-I & V model in *f(R)* theory of gravity.M.Sharif&H.Rizwanakausar[20] have discussed solution of Bianchi type-VI_0 universe by considering isotropic & anisotropic fluids in *f(R)* gravity. Reddy *et.al*[25] obtained the vacuum solution of Bianchi type-I & V models in *f(R)* theory of gravity under the special form of deceleration parameter. Adhav[26] obtained Bianchi type-III cosmological model filled with cosmic string within a framework of *f(R)* theory of gravitation.

The recent observations shows that our universe has undergoes from the several transition phase.These transitions may produce a string,which is form of topological defect.Due to this,string cosmological model received the praiseworthy attention from the researchers. One of the important event while universe is in transition phase is phase transition from quark to hadron which forms a strange quark matter.Related to strange quark matter, several researchers has been done study.

In the current study, we coupled a strange quark matter to the string cloud. It is fine to connect strange quark matter to the string cloud. Since the changes during the transition of universe could be quark-gluon plasma (QGP) hadron gas.In this model, we consider quarks are through as degenerate Fermi gas, which is exists only in arrange of space equipped with a vacuum energy density B_c (Known as Bag constant). When studying this model, the quark matter is composed of mass-less *u*&*d* quarks, massives quarks which are mass-less & non-interacting.

Therefore we have quark pressure for our model is

$$p_q = \frac{\rho_q}{3}\,, \qquad (1)$$

Where,$\rho_q$is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_c. \qquad (2)$$

But the total pressure is

$$p = \rho_q - B_c. \qquad (3)$$

Mak and Harko[27] has done the analysis of charged strange quark under the one parameter group . Dey*et.al.*[28]have derived the eqation of state for strange matter and described the mass – radius relation. The examination of strange quark matter attached to string cloud in the spherical symmetry modelhas been by Yavuz*et al*[29]. Adhav*et al*[30], [31]discussed string clouds and domain walls with quark matter in the N-dimensional Kaluza-Klein cosmological model and strange quark matter associated with the string cloud in Bianchi type III spacetime general relativity.Khadekar et al.[32]limited his work to quark matter related to topological errors in general relativity. Katore and Shaikh[33] , Rao and Neelima[34]obtained cosmological models with strange quark matter coupled with cosmic strings for axially symmetric space-time. Khadekar and Rupali[35]discussed the geometry of quark and strange quark matter in higher dimensional space-time in general relativity.Recently pawar*et.al* [36] studies string cosmological model with zero mass scalar field in *f(R)* gravity.

By inspiring with the above works,we study the anisotropic Bianchi type-V strange quark cosmological models in presence or absence of string cloud in frame work of*f(R)* modified theory of gravity. This paper is formulated as follows

In sect.2: Derivation of field equationof *f(R)* theory of gravity from concerning action.

Sect.3:We derive the field eq. of*f(R)* gravity for the Bianchi type-V metric in presence of strange quark matter attached to the string cloud.

Sect.4 is gives the solution of field eq. both in presence & absence of cosmic string.

In the last section we summarize & concluded the result.

2.Formulation of *f(R)* theory of gravity

The field equations of *f(R)* gravity are obtained from the action

$$S = \int \sqrt{-g}\left(\frac{1}{2k} f(R) + L_m\right) d^4x \quad (4)$$

Where, $k = 8\pi G$, *f(R)* is a general function of the Ricci scalar and $L_m$is the matter Lagrangian. For simplicity, k is taken as unity.

The divergence of activity (4) with respect to metric gives the following field equations:

$$F(R)R_{ij} - \frac{1}{2} f(R) g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \Box F(R) = T_{ij} \qquad (5)$$

Where $F(R) = \frac{df(R)}{dR}$and $\Box = \nabla^i \nabla_i$, ∇_i is the covariant derivative and $T_{ij}$is the energy momentum tensor of matter.

Apply $\delta_j^i = 4$and $\Box = \nabla^i \nabla_i$. The trace of field equation is

$$F(R)R - 2f(R) + 3\Box F(R) = T \qquad (6)$$

Applying (6) in (5),the field equations lead to the formation

$$F(R)R_{ij} - \nabla_i \nabla_j - kT_{ij} = g_{ij}\left(\frac{F(R)R - \Box F(R) - T}{4}\right) \qquad (7)$$

By diminish Eq.(5),we get

$$F(R)R_i^i - \frac{1}{2}f(R)\delta_i^i - \nabla^i \nabla_i F(R) + \delta_i^i \Box F(R) = T_i^i \qquad (8)$$

Equation (6) is an important relationship between *f(R)* and *F(R)* which will be utilized to simplify the field equations and to evaluate *f(R).*

3.Matric and the field Equation

We are consider the anisotropic Bianchi type-V metric of the form

$$ds^2 = dt^2 - A^2 dx^2 - e^{2mx}\,(B^2 dy^2 + C^2 dy^2) \qquad (9)$$

where*(x,y,z,t)* are Cartesian coordinate and A,B&C are function of cosmic time “t” only.

The energy momentum tensor for string cloud is given by

$$T_{ij} = \rho u_i u^j - \rho_s x_i x_j \qquad (10)$$

here$\rho$is the rest energy density and ρ_s is the string tension density. They are related by

$$\rho = \rho_p + \rho_s \qquad (11)$$

whereρ_p is the particle energy density.

If we are examined, the string has different vibration patterns representing different types of atoms, because these different patterns can be seen as different masses or spins. Therefore, in this case we want to use quarks instead of atoms in the string cloud. Therefore, in the language cloud we consider the energy density of the quark matter rather than the energy density of the atom.

In this case,from (11)we get

$$\rho = \rho_q + \rho_s + B_c, \qquad (12)$$

Where, $\rho_q$is quark energy density, Bc is bag constant and takes a value between 60 and 80$\frac{MeV}{(fm)^3}$.

From (11) and (12), we get energy momentum tensor for strange quark matter attached to the string cloud as

$$T_{ij} = (\rho_q + \rho_s + B_c\&)u_i u_j - \rho_s x_i x_j \qquad (13)$$

Where,x_i is the unit space like vector describing the direction of the string.

We have u_i and $x_i$with satisfying conditions:

$$g^{ij}u_i u_j = x^i x_j = 1 \text{ and} u^i x_j = 0 \qquad (14)$$

Here, we admit that the direction of string along the X-axis. So, we have

$$T_1^1 = \rho_s\,; T_2^2 = T_3^3 = 0 : T_4^4 = \rho \qquad (15)$$

By using co-moving coordinates, the field equation (8) for metric (9) generate the following series of equations:

$$F\left(\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{2m^2}{A^2}\right) - \frac{1}{2}f(R) + \left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{F} + \ddot{F} = \rho_s. \qquad (16)$$

$$F\left(\frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{2m^2}{A^2}\right) - \frac{1}{2}f(R) + \left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right)\dot{F} + \ddot{F} = 0. \qquad (17)$$

$$F\left(\frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{2m^2}{A^2}\right) - \frac{1}{2}f(R) + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{F} + \ddot{F} = 0.. \qquad (18)$$

$$F\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right) - \frac{1}{2}f(R) + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{F} = \rho. \qquad (19)$$

Where, Ordinary differentiation with respect to cosmic time 't' is represented by the overhead dot.

4.Solution of the field equations

4.1 Strange quark cosmological model of Bianchi type V with string cloud

From (16) to (19), we get the following equations:

$$F\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\frac{\dot{C}}{C}\right)+\left(\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\right)\dot{F}=\rho_s. \quad (20)$$

$$F\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{2m^2}{A^2}\right)+\left(\frac{\dot{C}}{C}\right)\dot{F}-\ddot{F}=\rho. \quad (21)$$

$$F\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}-\frac{2m^2}{A^2}\right)-\frac{1}{2}f(R)+\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{F}+\ddot{F}=\rho_p. \quad (22)$$

$$A^2 = mBC \quad (23)$$

where m is a constants of integration. The constant m, without loss of generality, can be chosen as unity [37] so that we have, from Eq. (23),

$$A^2 = BC \quad (24)$$

From Eqs. (20)–(23), We can see that we have three independent equations with seven unknown variables: *A, B,C, F(R)*, ρ_s , ρ_p and ρ. Therefore, To answer the above set of equations, we need some additional conditions.

Thus, we have considered the following physically valid assumptions:

1. The shear scalar σ^2 is proportional to scalar expansion θ so that we can take relationship between the metric potential *C& B*, given by Collins et al.

   $$B = C^n \quad (25)$$

   Where $n \neq 0$ is a constant.

2. Chiba et al.[38] studied that *f(R)* gravity is equivalent to gravity's scalar tensor theory of gravitation. Strange quark matter cosmological hypotheses are linked to the string cloud. However, Uddin et al.[39] have discover a result inthe context of *f(R)* gravity which that shows that

   $$F(R)\ \alpha\ (a(t))^k \quad (26)$$

   Where, k is an arbitrary constant.

   From Eq. (26), weget

   $$F(R) = F_0[a(t)]^k \quad (27)$$

   Where, F_0 is proportionality constant.

3. We consider hybrid expansion law (HEL) of the scale factor *a(t)*[40], given by

   $$a(t)=t^{\alpha}e^{t\beta} \quad (28)$$

   where, α and β are positive constants.

We use the above form of scale factor because it is the product of both exponential and power functions of cosmic time t, so the above form of the scale factor is more generic. The fundamental reason for selecting this scale factor is to observe in the model the smooth transition from early deceleration to late time inflation of the Universe. Also, this preferred average scale factor leads to a time dependent deceleration parameter. The average scale factor *a(t)* is defined as

$$a(t) = V^{\frac{1}{3}} = (ABC)^{\frac{1}{3}} \quad (29)$$

Now from Eqs. (24), (25) and (29), we obtain the metric potentials as

$$A = t^{\alpha}e^{t\beta} \quad (30)$$

$$B = (t^{\alpha}e^{t\beta})^{\frac{2n}{n+1}} \quad (31)$$

$$C = (t^{\alpha}e^{t\beta})^{\frac{2}{n+1}} \quad (32)$$

Hence, from Eqs. (20)–(22), we get the string tension density defined as

$$\rho_s = F_0\left(t^{\alpha}e^{t\beta}\right)^k\left[\left(\beta+\frac{\alpha}{t}\right)^2\left(\frac{3(1-n)+k(n-1)}{(n+1)}\right)+\alpha(n-1)(n+1)t2.\right. \quad (33)$$

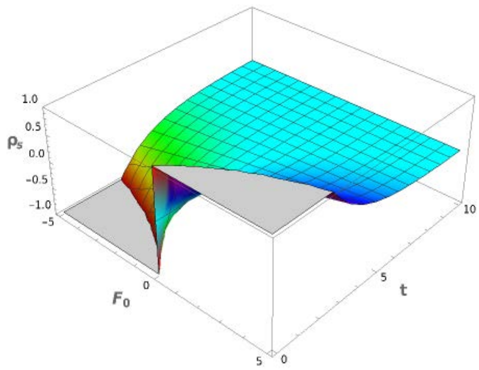

Fig.1 Plot of string tension density ρ_s versus cosmic time *t* for α=0.7, β=0.3, m=1,n=0.5, k=-2.

Fig.1 3D graph shows that behavior of string density (ρ_s) against time (*t*). Here, we observed that (ρ_s)is increasing function for $F_0 < 0$ and increasing $F_0 > 0$ with respect to cosmic time (*t)* throughout the evolution of the universe.

The string energy density is given by

$\rho = F_0\left(t^{\alpha} e^{t\beta}\right)^k \left[\left(\beta + \frac{\alpha}{t}\right)^2 \left(\frac{3n+2k-1}{n+1}\right) + \right.$
$\beta+$
$\alpha t22n(n-3)(n+1)2+\alpha-3n-1+k(n+1)(n+1)t2+2m2(t\alpha et\beta)-2-k2\beta+\alpha t2$. (34)

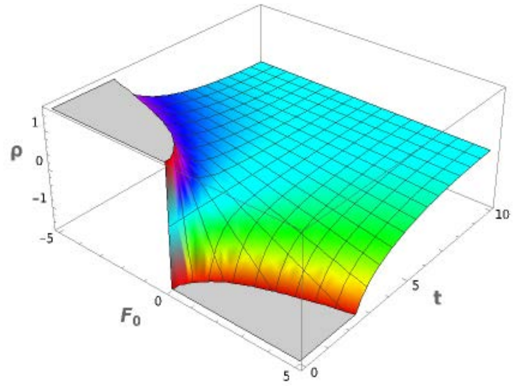

Fig.2 Plot of string energy density ρversus cosmic time t for α=0.7, β=0.3,m=1,n=0.5,k=-2.

From fig.2it can be seen that string energy density ρ approaches to zero for all values of F_0. This shows that, there is some secret energy known as dark energy.

Particle energy density defined as

$\rho_p = F_0\left(t^{\alpha} e^{t\beta}\right)^k \left[-k\left(\beta + \frac{\alpha}{t}\right)^2 - \right.$
$2n-12n+12\beta+$
$\alpha t2-2mt\alpha et\beta-2-\alpha 2-k1+nn+1t2+k2\beta+$
$\alpha t2$. (35)

Quark energy density is

$\rho_q = \rho - B_c$

$\rho_q = F_0\left(t^{\alpha} e^{t\beta}\right)^k \left[\left(\beta + \frac{\alpha}{t}\right)^2 \left(\frac{3n+2k-1}{n+1}\right) + \right.$
$\beta+$
$\alpha t22n(n-3)(n+1)2+\alpha n+1t2kn+1-3n-1+2m2t\alpha et\beta-2-k2\beta+\alpha t2-Bc$. (36)

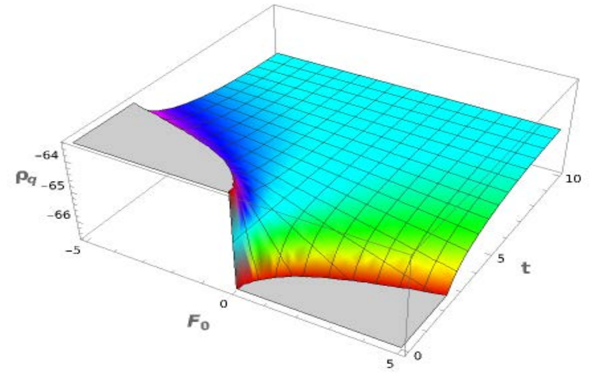

Fig.3 Plot of quark density ρversus cosmic time t for α=0.7, β=0.3, m=1,n=0.5,k=-2.

The quark energy density presented by 3D graph shown in fig.3 goes towards the constant value.

Quark pressure is given by

$p_q = \frac{\rho_q}{3}$

$p_q = \frac{F_0\left(t^{\alpha} e^{t\beta}\right)^k}{3}\left[\left(\beta + \frac{\alpha}{t}\right)^2 \left(\frac{3n+2k-1}{n+1}\right) + \right.$
$\beta+$
$\alpha t22n(n-3)(n+1)2+\alpha n+1t2kn+1-3n-1+2m2t\alpha et\beta-2-k2\beta+\alpha t2-Bc3$. (37)

Total pressure is defined as

$p = p_q - B_c$

$p = \frac{F_0\left(t^{\alpha} e^{t\beta}\right)^k}{3}\left[\left(\beta + \frac{\alpha}{t}\right)^2 \left(\frac{3n+2k-1}{n+1}\right) + \right.$
$\beta+$
$\alpha t22n(n-3)(n+1)2+\alpha n+1t2kn+1-3n-1+2m2t\alpha et\beta-2-k2\beta+\alpha t2-4Bc3$. (38)

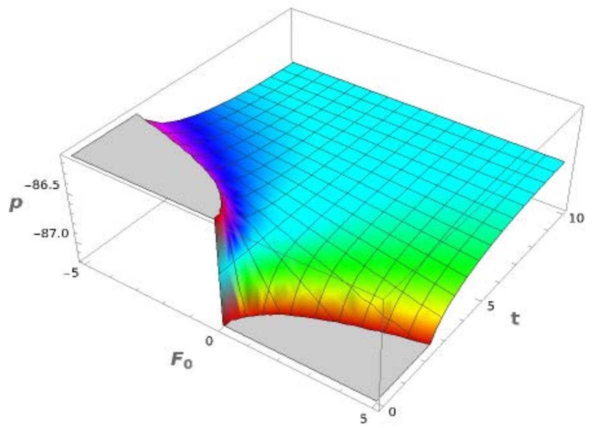

Fig.4 Plot of total pressure $p$versus cosmic time t for α=0.7, β=0.3,m=1,n=0.5,k=-2.

See the Fig.4, pressure is negative throughout evolution of the universe describe the phenomenon accelerated expansion of the universe in modified theories of gravity[41].From fig.2&4 energy density is zero in late time and pressure p is negative is an evident that expansion of the universe

The scalar curvature for the metric (9) is given by

$$R = 2\left[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{3m^2}{A^2}\right].$$

$$R = 2\left[5\left(\beta+\frac{\alpha}{t}\right)^2+\left(\beta+\frac{\alpha}{t}\right)^2\left(\frac{2(n^2+1)}{(n+1)^2}\right)-\right.$$
$3\alpha t2-\quad 3m2t\alpha et\beta-2.$ (39)

From Eq.(17), the function *f(R)* is given by

$$f(R) = 2\left[F\left(\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}-\frac{2m^2}{A^2}\right)+\dot{F}\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)+\ddot{F}\right].$$

$$f(R) = 2F_0\left(t^\alpha e^{t\beta}\right)\left[\left(\beta+\frac{\alpha}{t}\right)^2\left(\frac{6n}{n+1}\right)-\right.$$
$\alpha n+12n2+k+k-2m2t\alpha et\beta-2+ k2+3k\beta+\alpha t2.$ (40)

Now the metric (9) can be written as

$ds^2 =$

$$dt^2-\left(t^\alpha e^{t\beta}\right)^2dx^2-e^{2mx}\left(\left(t^\alpha e^{t\beta}\right)^{\frac{4n}{n+1}}dy^2+\right.$$
$t\alpha et\beta 4n+1dz2)$ (41)

4.1.1 Some important properties of the model

Equation (41) represents a four-dimensional anisotropic Bianchi type-V cosmological model with strange quark matter coupled to string cloud in *f*(*R*) gravity. Thefollowing physical and geometrical parameters are important in the discussion of our model. This model gives the idea about the evolution of ourUniverse.

1. The model's spatial volume (V) and average scale factor *a(t)* are given by

$$V=\sqrt{-g}=\sqrt{A^2B^2C^2}=ABC=\left(t^\alpha e^{t\beta}\right)^3 \quad (42)$$

And scale factor

$$a(t)=V^{\frac{1}{3}}=t^\alpha e^{t\beta}$$

2. The mean Hubble's parameter *(H)* is

$$H=\frac{H_1+H_2+H_3}{3}=\left(\beta+\frac{\alpha}{t}\right) \quad (43)$$

Where, $H_1=\frac{\dot{A}}{A}=\left(\beta+\frac{\alpha}{t}\right), H_2=\frac{\dot{B}}{B}=\frac{2n}{(n+1)}\left(\beta+\frac{\alpha}{t}\right), H_3=\frac{\dot{C}}{C}=\frac{2}{(n+1)}\left(\beta+\frac{\alpha}{t}\right)$ are the directional Hubble's parameters, which express the expansion rates of Universe in the direction of x,y& z respectively.

3. Anisotrpic Parameter $A_h$is

$$A_h=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2=\frac{2}{3}\frac{(n-1)^2}{(n+1)^2} \quad (44)$$

Where,$A_h$is the deviation from isotropic expansion and the Universe expands isotropically ifA_h = 0

4. Expansion scalar *(Θ)* is given by

$$\Theta=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=3\left(\beta+\frac{\alpha}{t}\right) \quad (45)$$

5. Shear scalar (*σ)* is given by

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^{\,2}-\frac{\theta^2}{3}\right)=\frac{(n-1)^2}{(n+1)^2}\left(\beta+\frac{\alpha}{t}\right)^2 \quad (46)$$

Where σ_{ij} shear tensor.

Fig. 4Shows the be deportment Hubble parameter *(H)* and expansion scalar (*Θ*) with respect to cosmic time (*t*). Both are tends towards constant value in a late times, as shown in following figure.

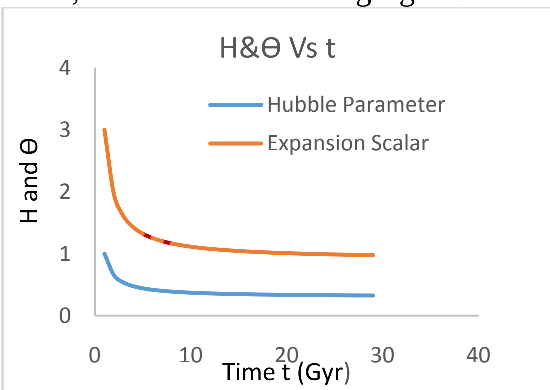

Fig.5 Plot of Hubbles's parameter H and expansion scalarΘ versus cosmic timet for α=0.7, β=0.3

6. The deceleration parameter of the model is defined as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{\alpha}{t^2\left(\beta+\frac{\alpha}{t}\right)^2} - 1 \qquad (47)$$

Recent observational data shows that a positive value of q is corresponding to the deceleration of the model, whereas a negative value of $q$shows acceleration and a universe has constant rate of expansion if q=0. According to recent researches, the currently our Universe is accelerating, and the value of the deceleration parameter is in the range $-1 \leq q < 0$.

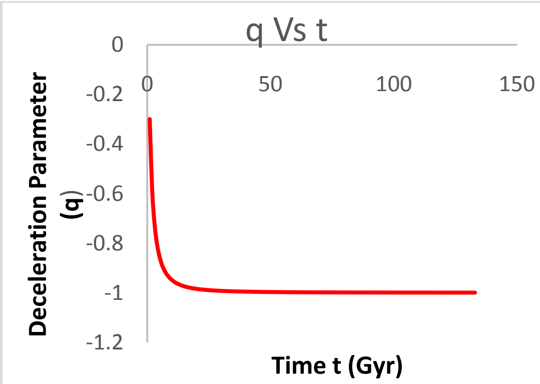

Fig.6 Plot of deceleration parameter versus cosmic time t (Gyr) for α=0.7, β=0.3.

Fig. 6 gives the idea of behavior deceleration parameter (q) versus time *(t)*. It is shows how the model is in current acceleration phase. The present value of the deceleration parameter(DP) is$q \approx -0.9$at $t \approx 13.8\, Gyr$ which is obtained from eq.(47) and which provides the additional evidence to the most recent observational result.[42]

7. In cosmology, the jerk parameter is defined as the dimensionless third derivative of a scale factor with respect to time and is which given by

$$j = \frac{\ddot{a}}{aH^3} = 1 + \frac{2\alpha}{(\alpha+\beta t)^3} - \frac{3\alpha}{(\alpha+\beta t)^2} \quad (48)$$

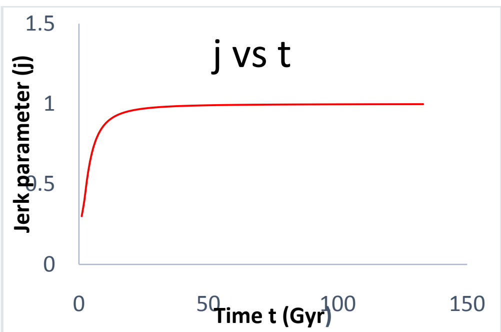

Fig.7 Plot of jerk parameter versus cosmic time t (Gyr) for α=0.7, β=0.3 .

It is a common assumption in current cosmology that the jerk parameter can describe the transition of the universe from a decelerating phase to an accelerating phase. This universe change occurs in several models with a positive jerk parameter and a negative deceleration parameter..[9], [43].

As illustrated in fig.6, we exhibit the curve for the jerk parameter versus cosmic time (t). We can observe that jerk parameter positive throughout the evolution of the universe and then reaches to constant value one. This demonstrates that our model is matches the observation with current observations.(i.e., ΛCDM models have a jerk parameter which is constant and equal to unity).

8. Spatial parameters like state finder parameters are constructed from space time metrics and used to analyze the geometric behavior of the universe. This defines dark energy more homogeneously than physical quantities, because the quantities in the model are dependent. As a result, higher-order derivatives of the scaling factor *a(t)* allow a more accurate analysis of space-time dark energy models.. For this purpose, a diagnostic proposal based on parameters pair {r,s} the so-called ‘‘state-finder’’, was introduced by Sahni et al.[44]. The state-finderpair {r,s} is defined as follows:

$$r = \frac{\ddot{a}}{aH^3} = 1 + \frac{2\alpha}{(\alpha+\beta t)^3} - \frac{3\alpha}{(\alpha+\beta t)^2} \qquad (49)$$

$$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = \frac{2\left((2\alpha-3\alpha(\alpha+\beta t)\right)}{3((2\alpha-3(\alpha+\beta t)^3)} \, (50)$$

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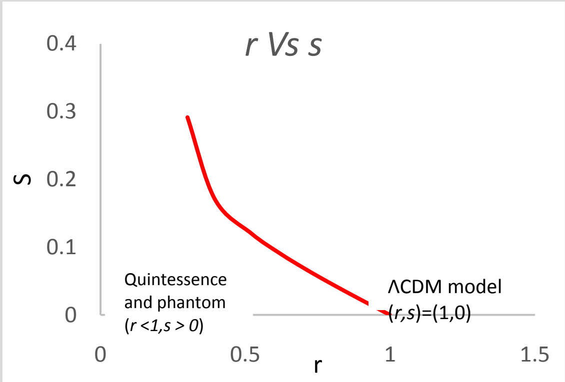

Fig.8 Plot of state-finder parameters (r Vs s) for α=0.7, β=0.3

Fig.8is the analysis of the state-finder parameters curve reaches the point (r,s)=(1,0), which is corresponding to the ΛCDM model. At certain point, our constructed model of the Universe behaves like a ΛCDM model, quintessence and phantom models for ($r <1,s > 0$).

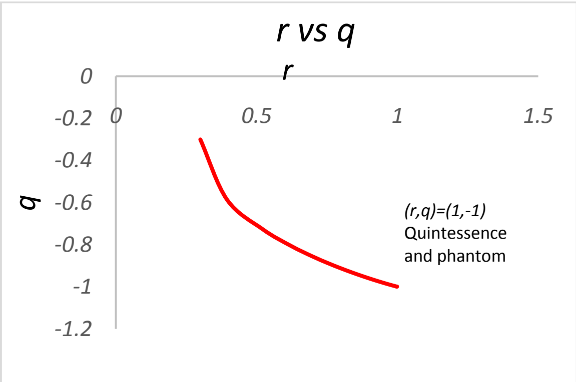

Fig.9 Plot of r vs q for α=0.7, β=0.3

Fig.9shows that the $r vs$ $q$parameters curve reaches the point (r,q)=(1,-1), the model is similar to quintessence and phantom model. This also a point which indicates that De-sitter expansion of the universe.

Discussion &Conclusion

In this study, we have investigated the Bianchi type –V cosmological model within a framework of $f(R)$ theory of gravity in presence of strange quark matter coupled with the one dimensional string.The field equation of our model has been solved by using hybrid expansion law & scale factor. In addition, we have studied several common cosmological parameters to examine their behavior our models. Our findings are reviewed as follows:

- From Fig.2&4, it is observed that pressure is negative and energy density tends to constant value zero which shows that accelerated expansion of the universe. From fig.1&3, String density reaches to zero and quark density is negative throughout the evolution of the universe which shows that conversion of matter into a dark energy.
- For our model, the spatial volume (V) initially at zero (i.e.at $t=0$) and then increases with cosmic time, which evident the volume of an expanding Universe. The plots of the Hubble's parameter (H) and the expansion scalar (Θ). H and Θ are seen to diverge at early time and reach to constant value at late time.The average anisotropic parameter A_h for model is constant and does not vanish, this shows that our model is anisotropic throughout evolution of the universe.
- The deceleration parameter (DP) for our model is negative throughout evolution indicates that our universe is in accelerated phase. The present value (i.e., at t_0=13.8 Gyr) of the DP is obtained as -0.9for$\alpha = 0.7, \beta = 0.3$, which matches the recent observational data of SNe Ia.
- We can see from Fig. 7 that the variation in jerk parameter versus cosmic time is positive indicates that the expansion of the universe is accelerating at an increasing rate.
- Fig. 8 shows the r-s plane corresponding to our model. It is observed that the built-in model of the universe initiates the development of the quintessence and phantom regions, progresses to the ΛCDM model (r=1, s=0)
- Fig.9$r vs$ q parameters curve reaches the point (r,q)=(1,-1), this point is indicates that De-sitter expansion of the universe.That is our universe as spatially flat and neglects ordinary matter and this is confirmed by Fig.1&2 which shows that the both string density ρ_s and energy densityρ approaches to zero for late timeacceleration of the universe. Finally, we can conclude that all the

above observations are in good evident with recent cosmological observations.

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