



WIRELESS SENSOR NETWORK LOCALIZATION USING SOCP RELAXATION

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Abstract— In recent year node localization is one of the most important issue since it plays a critical role in many situation. The aim of the sensor network localization problem is to determine positions of all sensor nodes in a network given certain pair wise noisy distance measurements and some anchor node positions using distributed localization algorithm based on second-order cone programming relaxation. The distributed sensor network localization algorithm using SOCP Relaxation is implemented and Simulation for uniform network topologies shows, the anchor position and distance estimation errors, and the performance gains achievable in terms of localization accuracy and computational efficiency.

Index Terms— Distributed algorithms, convex optimization, relaxation methods , Localization

I. INTRODUCTION

The development of MEMS, chip systems and wireless communications technology has fostered, low-powered and multi-function sensor nodes, which can integrate information collection, data processing, wireless communications and other functions together within the small storage, to gain rapid progress [1]. WSN is a multi-hop self-organizing network, where a large number of sensor nodes are deployed. The aim of WSN is to perceive, collect and process the information of sensor nodes within the coverage of the network [1].

In wireless sensor networks (WSN's), localization is often performed by using the information of time-of-arrival (TOA), time-difference of-arrival (TDOA), received-signal-strength (RSS) measurements, or a combination of them [2]. However, localization by using TOA or TDOA information (including the Global Positioning System (GPS)) requires the complicated timing and synchronization, which makes sensor node localization cost-expensive and is not suitable for sensor networks with small, simple and cheap nodes [9]. In indoor environments, the signal from the GPS satellites is too weak to penetrate most buildings, making GPS useless for indoor localization [4]. We here consider the problem by using the RSS information without the need of timing and synchronization.

Currently, most node localization schemes for WSNs are relying on a small fraction of beacons scattered throughout the sensor network.. Beacons are sensor nodes which know their own positions (through GPS or other manual configurations) and serve as a reference for other nodes whose positions are unknown as they are deployed. The position discovery for the unknown nodes in these cases intends to exploit the multi hop character of WSNs, and rely more on the node-to-beacon distance measurements. Moreover, due to some special limitations on the sensor nodes, such as low memory and bandwidth, short battery life, and limited communication and computation capability, a node localization scheme is commonly required

distributed, robust and energy efficient, and so on [5].

In this paper, we present a distributed algorithm based

on second-order cone programming (SOCP) for solving the sensor network localization problem. In the presence of distance estimation errors each sensor node determines its position by executing the localization algorithm independently using distance information to the anchor and sensor nodes with which it is directly linked (i.e., which are within its communication range). If in addition to the distance estimation errors, the anchor positions also have errors then the algorithm consists of three steps: using the local distance information and inaccurate anchor positions each sensor node estimates its position. Then, the anchors execute the localization algorithm using position information from their neighbouring nodes and the associated distance information to refine their positions. Finally, the sensor nodes execute the localization algorithm to refine their position estimates [9].

One of the significant advantages of our approach is that it is fully distributed and converges to an optimal (or near-optimal) solution. As a result of the distributed nature of the solution, the problem dimension at each node is a linear function of only the number of neighbours of the node. There is no significant increase in the computational effort per node even in large networks (for a given node connectivity level), whereas most existing methods result in an exponential increase in the computation time with network size gets reduced. Thus, the distributed SOCP approach is suitable for large-scale networks with thousands of nodes.

The rest of the paper is organized as follows. Section II provides an overview of existing approaches. Section III presents the mathematical formulation and the SOCP relaxation of the localization problem. In section IV we present the distributed localization algorithm based on the SOCP relaxation. The simulation study appears as sections V.

II. RELATED WORK

A more generalized form of the localization problem is the distance geometry problem. It has been studied extensively, mostly in the framework of Euclidean distance matrix (EDM) completion. Schoenberg and Young and Householder [5] established some basic

properties of distance matrices. Ideas presented therein form the basis of a class of algorithms known as Multidimensional Scaling. Algorithms based on MDS sometimes use objective functions (such as the STRESS criterion) that ensure low-dimensional solutions for the given incomplete distance matrix.[22] The problem with these techniques is that there is possibility of getting stuck in local minima due to non-convex nature of the problem, and there is no guarantee of finding a desired realization in polynomial time. apply distributed weighted MDS (dwMDS) to the localization problem and formulate the problem using a general form of the cost function we use in this paper. They solve the minimization problem using majorizing functions. Biswas and Ye [22] solve the problem using the semidefinite programming (SDP) relaxation. This approach can solve small problems effectively. The authors report a few seconds of PC execution time for a 50 node network. They have also proposed two techniques to improve the accuracy of the SDP solution. The first technique adds a regularization term to the objective function to force the SDP solution to lie close to a low dimensional subspace of R^d and the second technique improves the SDP estimated solution using a gradient-descent method. [23]

A few variations of the original problem include localization in NLOS sensor networks, with mobile sensors, etc. Sensor Localization forms a sub-problem of the larger set of Graph Realization Problems. Other problems including, but not limited to molecule structure prediction, data visualization, internet tomography and map reconstruction. The concept can also be extended to problems of dimensionality reduction. Examples include face recognition and image segmentation.

A. Computational Complexity of Sensor Localization

The problem with MDS based algorithms is that there is a possibility of getting stuck in local minima due to the nonconvex nature of the problem, and there is no guarantee of finding a desired realization in polynomial time. With this in mind, researchers have also attempted to pin down the exact computational complexity of this problem. Saxe proved that the EDM problem is NP-hard in general. See also [2]. Aspnes et al.[3] proved a similar result in the context of sensor localization. More and Wu, who used global optimization techniques for distance geometry

problems in the molecule conformation space, established that finding an optimal solution, when is small, is also NP-hard. Other general global optimization approaches which employ techniques like pattern search face similar problems.

Localization as such is a non-convex optimization problem with multiple local minima. It was formulated as a feasibility problem with convex radial constraints by Dohery et. al. [3]. However, it required centralized computation which made it unsuitable for large-scale networks. A distributed localization method MDS MAP(P,R) was proposed by Shang based on multi-dimensional scaling (MDS) [2]. However, it involved lots of redundant calculation while merging local data for sensors to get global data for the entire network.

The SDP relaxation problem proposed by Biswas and Ye [24] adds a regularization term in the objective function to reduce the rank of the SDP solution, thereby reducing the estimation error. Refining of the initial estimates is also done, using a gradient-descent method.

Computational efficiency becomes increasingly important in case of mobile sensor networks, requiring dynamic estimation of sensor positions.

III.SENSOR NETWORK LOCALIZATION: PROBLEM FORMULATION

Sensor nodes measure physical quantities at a given position. In most applications, the data reported by the sensors is relevant only if tagged with accurate position of the nodes. But equipping each node with a GPS is a costly affair. Also, it has geographical constraints (for instance, it doesn't work indoors). Hence, the sensor network localization problem is of extreme importance.

It can be formulated as follows:

“Assuming knowledge of the positions of some nodes and some pairwise distance measurements, determine the position of all nodes in the network. Nodes whose positions are known beforehand are called reference nodes (RN) or anchors, and nodes whose positions are unknown as the un-localized nodes (UN) or sensors. The localization problem can be broken down into two sub problems:

(i) Ranging: To determine the distance (or range) between two neighboring nodes, for select nodes, depending on the model used. Usually, the

constraints are noise in measurement and non-practicality of large distances between nodes, hence distances only less than a specified “RadioRange” are considered.

(ii) Positioning: To determine the position or location of the nodes given some pairwise distances.

Mathematically speaking, there are distinct points in R^d ($d \geq 1$). We know the Cartesian coordinates of the last $n-m$ points (“anchors”) x_{m+1}, \dots, x_n and the Euclidean distance $d_{ij} > 0$ between “neighboring” points i and j for $(i, j) \in A$, where $A \subseteq (\{1, \dots, n\} \times \{1, \dots, m\}) \cup (\{1, \dots, m\} \times \{1, \dots, n\})$. We wish to estimate the Cartesian coordinates of the first m points (“sensors”).”

Here, we present a systematic approach towards sensor localization taking into account the statistical modeling of the ultra-wideband physical layer channel. This is accomplished through a distributed approach to refine sensor position estimates. Because most sensor localization approaches in the literature do not take into account the errors in node positions. But here, we have assumed erroneous node positions (both for sensors and anchors) and the localization is done in a three step process.

(1) Sensor positions are estimated using information from their neighbors.

(2) Anchor positions are refined using relative distance information exchanged with their neighbors.

(3) Sensor positions are re-estimated using refined anchor positions of their neighbors.

Such a distributed approach goes a long way in discarding the effects of inaccurate node positions.

Simulations have performed on uniform and irregular network topologies, and dependency of localization accuracy and computational efficiency with various factors has been studied.

A. Convex Optimization using Matlab

MATLAB takes the help of certain “solvers” for solving convex optimization problems; common ones include CVX, SEDUMI, CPLEX, GUROBI etc. Different solvers are recommended for different optimization problems. E.g. SEDUMI, SDPT3, CSDP, SDPA work best for Semidefinite Programming (SDP), while GUROBI and MOSEK are good for integer linear programming (LP). Now, which solver to choose depends on both the problem size and the type of the problem.[26]

The variations in the type of problem can include constraints, which can be one the 5 types:

- None(unconstrained)
- Bound
- Linear (including bound)
- General (smooth)
- Discrete (integer)

As well as the type of optimization problem to be solved, which can be of one of the following types:

- Linear
- Quadratic
- Sum-of-squares (Least Squares)
- Smooth Nonlinear
- Non-smooth

IV. DISTRIBUTED SOCP LOCALIZATION ALGORITHM

1) SOCP Relaxation

SOCP has been chosen due to its simpler structure and computational efficiency. The SOCP relaxation was first studied by Tseng. Although it is weaker than SDP, its computational superiority enables the use of localization in mobile sensor networks.

In general, a second-order cone program (SOCP) is defined as:

$$\begin{aligned} &\text{minimize } f^T x \\ &\text{subject to } \|A_i x + b_i\|_2 \leq c_i^T x + d_i, \quad i=1, \dots, m \\ &Fx = g, \end{aligned}$$

where $x \in R^n$ is the optimization variable, $A_i \in R^{n \times n}$, and $F \in R^{p \times n}$,

Now, the problem can be formulated as

$$\min_{x_1, \dots, x_m} \sum \left\| \|x_i - x_j\|^2 - d_{ij}^2 \right\| \quad (1)$$

where $\| \cdot \|$ denotes the Euclidean norm.

This can be rewritten in a convex form (by relaxing equality constraint to “greater than or equal to” inequality) as:

$$\min_{x_1, \dots, x_m} \sum_{(i,j) \in A} |y_{ij} - d_{ij}^2| \text{ s.t. } y_{ij} \geq \|x_i - x_j\|^2, \text{ for all } (i,j) \in A \quad (2)$$

The SOCP has $(d+3)|A| + m \cdot d$ variables and $(d+2)|A|$ equality constraints. In sensor network localization, $|A| = \Omega(m)$ and $d = 2$, so that (7) has $\Omega(m)$ variables and $\Omega(m)$ equality constraints.

Distributed Algorithm:

In a distributed algorithm, the optimal result is obtained in stages. Timing of computations at any one processor/node during a stage can be independent of the timing of computations at nodes in the same stage. All interactions and exchange of information / refining of positions takes place at the end of a particular stage. Here, in the SOCP it has been found in paper that each sensor can independently solve the minimization problem using position information only from its neighbor nodes.

Let $NB_A(i)$ denote the set of neighbor nodes for node x_i of the network. Above SOCP can be solved independently over the m sensor nodes, where each node uses information (x_j, d_{ij}) from its neighboring nodes $x_j, j \in NB_A(i)$. Thus, the SOCP decomposes to the following distributed formulation:

$$\min_{x_i, y_{ij}, t_{ij}} \sum_{j \in NB_A(i)} t_{ij} \quad (3)$$

$$\begin{aligned} \text{s.t. } &y_{ij} \geq \|x_i - x_j\|^2, \quad \text{for all } (i,j) \in A \\ &t_{ij} \geq |y_{ij} - d_{ij}^2| \end{aligned}$$

The distributed SOCP algorithm consists of a phase where each sensor node estimates its position using local information and solving the SOCP (3). In an iterative distributed scheme, this would be followed by a communication phase wherein each node exchanges its position estimate with its neighbors. These iterations are repeated after fixed intervals of time or when any new information becomes available at a node. It should be noted that the algorithm uses information from neighboring anchors as well as sensors to position a given node. Thus to obtain a non-trivial position estimate each node needs at least 3 neighbors (for 2-D localization) with position estimates, as opposed to the more stringent requirement of having 3 anchors in the neighborhood that many triangulation/trilateration schemes impose. If the anchor positions are inaccurate, the distributed SOCP approach will consist of three steps: after the sensor nodes estimate their positions based on the inaccurate anchor positions and distance information, the anchors solve the local SOCP using position information from their neighboring nodes and the associated distance information to refine their positions. As we will show, this second step results in a significant improvement in the positioning accuracy of the inner anchors. Finally, another iteration of the

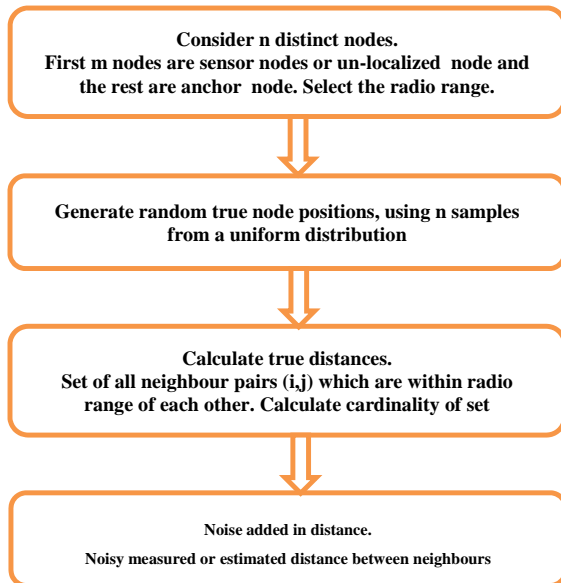
local SOCP over the sensor nodes further refines their position estimates.

Let $n_i(=|N_A(i)|)$ represent the number of neighbors of the node x_i . SOCP has $2n_i+3$ variables, $2n_i$ conic constraints and 1 equality constraint. In sensor networks, due to the short radio range of the sensors, the number of neighbors of a given node is a small fraction of the total number of nodes in the network (i.e., $n_i \ll n$). Thus, the distributed SOCP approach results in significantly smaller problem sizes than approaches proposed in the literature. The SOCP problem can be efficiently solved by interior point methods. Here we use SeDuMi to solve this problem.

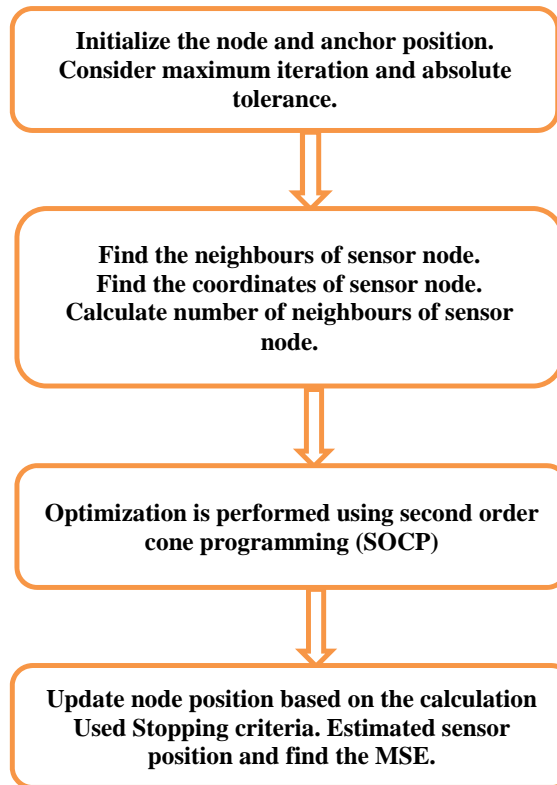
V. Algorithm and Simulation results

1. Distance Measurement Algorithm:

In this algorithm using anchor node and pair wise distance is measured. Position computation algorithm using this distance find the node position. finally find the error between true node position and estimated node position.



2. Position Computation and Localization Algorithm:



B. Simulation Results :

Simulation is performed in matlab. In this experiment using uniform distribution is used for node deployment. As shown in figure.1. number of nodes is 300, number of node is 0.15 percentage, RadioRange 0.06 and noise factor n_{fa} is 0.05. In figure 1 sensor node shown by anchor node shown by \times and estimated position of node is denoted by $+$. Line shows the true node position and estimated position difference.

fig.1. Distributed SOCP results for Uniform topology: $n = 300$, Radio Range = 0.6, $p = 0.15$ and $n_{fd} = 0.05$

Test case	n	Radio Range	p	CPU time per node (in sec)	Error
1	500	0.10	0.15	0.2188	0.0791
2	500	0.12	0.15	0.2500	0.1292
3	500	0.14	0.15	0.2836	0.0256
4	500	0.16	0.15	0.3438	0.0345

5	50 0	0.18	0.15	0.2344	0.04 04
6	50 0	0.20	0.15	0.3906	0.03 92
7	50 0	0.22	0.15	0.4688	0.03 91
8	50 0	0.24	0.15	0.4844	0.03 33
9	50 0	0.26	0.15	0.4688	0.04 27

Table 1 . Different RadioRange for n=500,p=0.15

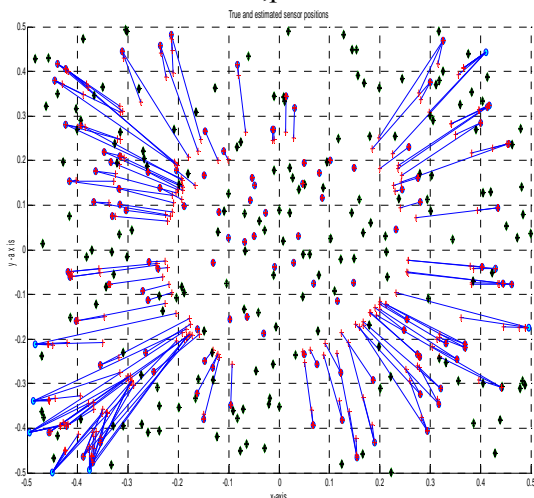


fig.1. Distributed SOCP results for Uniform topology: n = 300, Radio Range = 0.6, p = 0.15 and nfd = 0.05

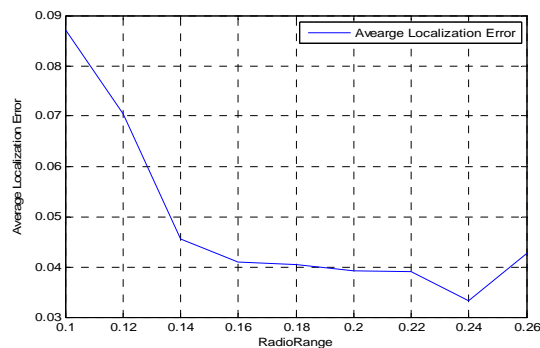


Fig.2. Average positioning error as a function of different RadioRange. (n = 500,p= 0.15 and nfd= 0.05)

Test	n	p	nfd	Radio	CPU Time per node	Error
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case				Range		
1	100 0	0.1 5	0.0 5	0.08	0.2969	0.04 97
2	100 0	0.1 5	0.0 5	0.10	0.2656	0.02 98
3	100 0	0.1 5	0.0 5	0.15	0.3438	0.02 81
4	100 0	0.1 5	0.0 5	0.20	0.6719	0.02 68
5	100 0	0.1 5	0.0 5	0.25	0.8125	0.02 94

Table.2 Different RadioRange for n=500,p=0.15

Fig.3. Average positioning error as a function of different RadioRange. (n = 1000,p= 0.15 and nfd= 0.05)

COMPARISON

Plot the graph noise factor and average node localization as shown in fig.4. Noise factor is increased average node localization accuracy is decrease. This graph shows that changes in distributed SOCP algorithm in k and pars. structure gives less error as compare to SOCP algorithm. Table shows parameter of the distributed SOCP algorithm. The performance increases is shown in graph and table.

As shwn in table implemented SOCP can give compare to paer (9). Taken 500 node , RadioRange 0.15, anchor percentage 0.15, noise factor for sensor node is difeerent different and noise factor for anchor node is taken 0.10 and simulate.

Test case	n	RadioRange	p	nfa	nfd	Impl emented SOCP error	SOC P error
1	50 0	0.15	0.1 5	0.1 0	0.0 2	0.04 06	0.04 80
2	50 0	0.15	0.1 5	0.1 0	0.0 4	0.04 11	0.04 90
3	50 0	0.15	0.1 5	0.1 0	0.1 0	0.04 34	0.04 70

4	50 0	0.15	0.1 5	0.1 0	0.1 5	0.04 83	0.05 35
5	50 0	0.15	0.1 5	0.1 0	0.2 0	0.05 22	0.05 90

Table 5.3 Different noise factor for sensor node nfa and its parameter for n=500, RadioRange=0.15,p=0.15 and anchor node position nfa=0.10

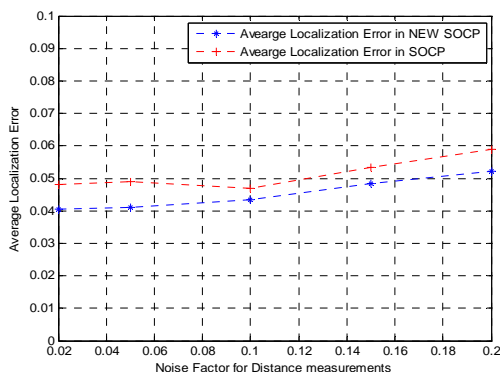


Fig. 4. Average positioning error as a function of the Noise Factor nfd. (n = 500, Radio Range = 0.15, p = 0.15)

CONCLUSIONS

Distributed approach goes a long way in discarding the effects of inaccurate node positions. Simulations have performed on uniform topology, and dependency of localization accuracy and computational efficiency with various factors has been done. Average node localization accuracy is improved as compared to the Distributed SOCP localization accuracy. In SeDuMi solver a Different parameter of second order cone constraints are used and get accurate result.

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